The Application of Mathematics Learning Model to Stimulate Mathematical Critical Thinking Skills of Senior High School Students

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Abstract: The objective of this research is to analyze the twelfth graders' mathematics critical thinking skills using a mathematics learning model to stimulate fundamental critical thinking abilities of science courses in SMA Negeri, Pacitan Regency, East Java Province, Indonesia. This quasi-experimental research design was used in this study with one group posttest only design using multiple substantive posttests. The sample of 141 students from the total population of six public schools involving the twelfth graders of the natural sciences was selected through purposive sampling technique, data were taken through tests of students' critical thinking skills and interviews. The data analysis consists of five stages, namely an analysis of one sample t-test, an analysis of students' grades, an analysis of problem-solving stages, an analysis of critical thinking abilities indicators, and an analysis of mathematics critical thinking abilities indicators. The results showed (1) The results of the one sample t-test show that the mathematics learning model is effective to stimulate critical thinking, which means that the application of the mathematics learning model is effective to stimulate critical thinking; (2) the overall grades of students that met the minimum mastery criteria; (3) the data analysis of eleven problem-solving stages proves that the criteria for critical thinking abilities are categorized as good and very good. The highest score indicator considers the principle and definition of transformation, while the lowest grade indicator is mainly concerned with the questions on right and coherent steps; (4) the critical thinking skills have seven indicators that highlight the criteria of students' critical thinking abilities categorized as good and very good. The indicators that get the highest score determine the definitions of terms, while the indicators of the lowest score determine the action; (5) the results of the analysis show indicators of mathematics critical thinking skills that have eight indicators. The criteria of students' critical thinking abilities met good and very good categories along with indicators with the highest value score by considering the definitions of terms, while the indicators of the lowest score deal with the habit of caution.

Keywords: Analysis, application of learning models, critical thinking skills.

Introduction

Indonesia has ranked at the bottom of the list in terms of the Human Development Index (HDI) for several years. Indonesia's 2014 HDI was 110th out of 188 countries with a HDI value of 0.684 (United Nations Development Programme [UNDP], 2016). However, there has been an increase in 2019 where Indonesia has joined the ranks of countries in the world with high human development (UNDP, 2019). Here, it is very clear that Indonesia is serious about improving the quality of human resources (HR).

Although there is a very encouraging increase, in the education sector there is still much that needs to be improved. Different educational problems in Indonesia hinder the achievement of educational goals as mandated in Law No 20 the Year 2003 concerning the national education system. The current low quality of Indonesian human resources results from the low quality of education (President of Indonesia, 2003). This can also be viewed from wide-ranging micro indicators whose results show that Indonesian students have dissatisfactory performance resulting in students' low critical thinking skills.
Some studies from international institutions show that the mathematical abilities of Indonesian students are still low compared to other countries. One of the international studies is the Trends in International Mathematics and Science Study (TIMSS), an international study that evaluates existing education; especially the students’ learning outcomes aged 14 years at the junior high school level. A study by TIMSS on 2015 shows that Indonesia ranked 45th out of 50 participating countries (Mullis et al., 2015).

TIMSS introduces four levels of students’ abilities, namely low, medium, high and advanced. The results of the TIMSS study were not good because students were not able to solve TIMSS questions on medium, high and advanced criteria. Instead, they can only master simple mathematics or easy problem-solving, but have not been able to solve two problems, complex problems, and complicated problems. For this reason, the mistakes questions include non-routine questions that need further thoughts to solve problems (Phillips & Phillipov, 2005). In other words, the ability to think at the higher level and critical thinking skills are necessary to figure out problems (Sanders & Moulenbelt, 2011).

Indonesian students are not trained to work on non-routine questions (Ahmatika, 2017). Learning is dominantly connected to simple questions, in the sense of questions that have been solved clearly, namely single-ended rather than open ended questions. It can be a routine problem, which is a matter of ordinary practice to set procedures of learning in the class (Mwei, 2017).

In Indonesia, the ability to High Order Thinking (HOT) in mathematics is not optimally developed (Yuliati & Lestari, 2018). As a result, Indonesian students cannot think critically to identify new problems, search for and develop materials or ideas to solve them, let alone have to flexibly use settlement procedures (Liberna, 2012). Firdaus et al., (2015) mentioned that students are less able to think critically because they prioritize the process of memorizing, understanding and focus on memorizing concepts.

Critical thinking skills become a basic capital or intellectual capital for everyone. Therefore, critical thinking skills are crucial for students at every level of education (Howard et al., 2015). Schools are required to provide education that is able to foster student character to think critically, creatively, be able to communicate, and collaborate, so that students can adapt to the 21st century. This is in line with the 21st century 4C competencies that students must have, namely Critical Thinking and Problem Solving, Creativity, Communication Skills, and Ability to Work Collaboratively (Zubaidah et al., 2015). Things that are not much different are expressed by Kivunja (2015) which states that creative, critical thinking, independent, able to work with teams, creativity, information, communication and independent learning are competencies that must be mastered by every individual to face 21st century global competition.

Critical thinking skills are one of the life skills that students must have. Having critical thinking skills will help students solve simple or complex problems. Critical thinking allows students to discover the truth and process information logically so that they can determine which information is important, irrelevant, or useless (Antika, 2017; Amin et al., 2019). Critical thinking skills are needed by individuals to face various problems faced in social and personal life (Nuryanti et al., 2018).

Ennis explained critical thinking is a process that aims to make decisions so that what people think is the best about a truth and they can do things correctly (Ennis, 1993). Critical thinking is an important skill that helps students improve their ability to make judgments, inform well, explain reasons, and solve unknown problems (Facione, 1991). More over Masek and Yamin (2012) mention that critical thinking is the ability to analyze and evaluate information and ask important questions. Critical thinking leads to valid arguments and conclusions, strong and resistant criticism.

To overcome the students’ low critical thinking skills, many studies have been conducted previously by developing various methods (Aslan & Aybek, 2019; Kardoyo et al., 2019; Sasi, 2018). Setiana et al. (2019) in his previous research has also developed a learning model commonly called a mathematics learning model that stimulates critical thinking skills. The learning model has several stages, namely: 1) Providing a simple explanation (elementary clarification) that focuses on questions, analyzing questions and asking questions, and answering questions concerning an explanation or statement; 2) Building basic skills (basic support) by considering whether or not the source can be trusted and by analyzing an observation report; 3) Summing up (interference), namely the activity of reducing or considering the results of deduction, inducting or considering the results of induction, making and determining the value of consideration; 4) Providing advanced clarification, namely identifying terms and definitions of consideration and dimensions, and identifying assumptions; 5) Managing strategies and tactics, namely determining the actions and interacting with others.

Furthermore, Setiana et al. (2019) suggested that the learning model stages were combined with stimulation stages in the form of a) reinforcement, b) module usage, c) test provision, d) student activity, e) without punishment, f) direction, g) giving feed back. Setiana et al. added that the occurrence of critical thinking in learning mathematics is to present non-routine and open-ended contextual problems both individually and in groups by utilizing students’ initial knowledge (Setiana et al., 2019; Romberg, 1995).

The main critical thinking theory in this research is the development of critical thinking proposed by Ennis (Ennis, 1993) through six critical thinking elements synchronized into the acronym FRISCO (Focus, Reason, Inference,
Situation, Clarification, and Overview). The stimulation stages of critical thinking skills synthesized from a theoretical study consist of 5 components and 12 indicators and 27 sub-indicators as set out in the table below.

<table>
<thead>
<tr>
<th>No</th>
<th>Phases</th>
<th>Indicators</th>
<th>Sub-indicators</th>
</tr>
</thead>
</table>
| 1  | Providing a simple explanation (Focus) | Focusing on the questions | a. Identifying the questions  
b. Maintaining the condition of thinking  
c. Identifying conclusions  
d. Identifying interrogative sentences  
e. Reviewing the structure of an argument |
|    |        | Analyzing the arguments | a. Providing a simple explanation  
b. Citing examples |
|    |        | Asking questions, answering questions, clarifying challenging questions | a. Identifying conclusions  
b. Identifying interrogative sentences  
c. Reviewing the structure of an argument |
| 2  | Building basic skills (Reason) | Considering whether or not the source can be trusted | a. Considering the suitability of sources  
b. Considering the use of appropriate procedures  
c. The ability to give reasons  
d. Careful habits |
|    |        | Observing, considering the results of observations | a. Reporting the results of observations  
b. Accountability for the results of observations |
|    |        | Deducing, considering the deduction | Conditioning the logic |
|    |        | Inducing, considering the results of induction | a. Putting forward general things  
b. Expressing conclusions and hypotheses  
c. Drawing conclusions from the results of investigation |
|    |        | Making and reviewing the values of the results as highly considered | a. Making and determining the results of based on background facts  
b. Making and determining the results considered as problems |
| 3  | Concluding (Inference) | Defining terms, considering definitions | a. Making a definition form  
b. Acting by providing further explanation  
c. Explanation is not a statement  
d. Constructing an argument |
|    |        | Identifying assumptions | a. Choosing the criteria to consider possible solutions  
b. Formulating alternative solutions  
c. Using arguments  
d. Using logical strategies |
| 4  | Providing further explanation (Situation and Clarification) | Determining the action | a. Choosing the criteria to consider possible solutions  
b. Formulating alternative solutions  
c. Using arguments  
d. Using logical strategies |
| 5  | Setting strategies and tactics (Overview) | Interacting with other people | a. Using arguments  
b. Using logical strategies |

Learning mathematics is mainly concerned with how a mathematics learning model stimulates the students' critical thinking skills to solve problems (Setiana et al., 2019). Students who can think critically can evaluate their thoughts and compare them with data, facts, opinions, and thoughts from others (Walker, 2003). Starting from the previously elaborated facts, further research was conducted to analyze the students' mathematical critical thinking skills after implementing a mathematics learning model to stimulate critical thinking skills of the twelfth grade students at state senior high schools in Pacitan Regency, East Java, Indonesia.

Methodology

Research Design

The quasi-experimental research design was used in this study with one group posttest only design using multiple substantive posttests, where measurements of the research subject were only carried out after treatment using many substantive post-treatment measurements (Hastjarjo, 2019). The research design can be visualized in Figure 1.

![Figure 1. The research design](image-url)
This study used one group of subjects as an experimental group. In this study, the treatment used is the implementation of a mathematics learning model to stimulate critical thinking. After the treatment, several types of data were analyzed.

Research Setting

This research was conducted in six state high schools in Pacitan Regency, East Java, Indonesia from November 2016 to January 2017. The six high schools are SMA N 2 Pacitan, SMA N Punung, SMA N Tulakan, SMA N 1 Ngadirojo, SMA N 2 Ngadirojo, and SMA N Tegalombo.

Research Questions

1. How is the ability to think critically in terms of student scores after the implementing a mathematics learning model to stimulate critical thinking skills?
2. How is the ability to think critically in terms of problem solving stages after the implementing a mathematics learning model to stimulate critical thinking skills?
3. How is the ability to think critically in terms of critical thinking indicators in general after the implementing a mathematics learning model to stimulate critical thinking skills?
4. How are students' mathematical critical thinking skills based on mathematical critical thinking indicators after the implementing a mathematics learning model to stimulate critical thinking skills?

Participants

The research population included all students of twelfth grades in such state high schools as of the 2016/2017 odd semester. The samples in this study were one class XII IPA-1 each from six high schools in Pacitan Regency, totaling 141 students. The sample was selected by purposive sampling technique with the consideration that students in the IPA-1 group had a better level of competence than other classes because the critical thinking learning target must be adjusted to students who do have that potential. The respondents as interview sources were two students from each school, so that there were 12 people in total.

Data Collection Tools and Research Instrument

The retrieval of data uses the test techniques, i.e., tests of mathematics critical thinking skills and interview. The mathematics critical thinking abilities test was carried out at the end of the research to determine the level of students' mathematics critical thinking abilities after applying the mathematics learning models to stimulate critical thinking skills. Based on the data collection technique, the research instrument was a test instrument. The test instrument has been validated by two experts with the results showing valid criteria so that it is suitable for use in research. The validity test was also carried out using Product Moment Pearson Correlation obtained 5 valid questions. By using Alpha Cronbach's, the test reliability calculation results were 0.906 > 0.70 so that the test reliability was high. A test instrument in the form of essay questions with five open-ended test questions on the geometry transformation material. What is meant by open-ended questions is that one question can be solved in several ways, in this case students are asked to complete with the graphical method and the substitution method, and a question "what if ...". Each item must involve a coherent manner consisting of eleven stages developed through critical thinking indicators to solve problems.

The results of the test analysis were then strengthened by in-depth interviews. Interviews are used to verify student answers based on aspects of students' critical thinking abilities. For the focus aspect, students are expected to be able to convey the information and questions that are meant in the questions. In the reason aspect, students are able to explain the reasons relevant to each step of solving the problems they write. In the inference aspect, students are able to provide reasons for written conclusions. In this aspect of the situation, students are able to distinguish between information that is important and needed in solving problems with information that is less important. In the clarify aspect, students are able to make examples of similar questions with questions that have been solved verbally. Aspect overview shows students examine the results of work by relating to the context of the problem that has been solved. The interview subjects were two students from each school.

Data Analysis

In accordance with the predetermined research design namely quasi-experimental research design with one group posttest only design using multiple substantive posttests, after giving the treatment a substantive posttest measurement is carried out. Therefore, the test result data were analyzed based on five types of analysis methods. The analysis involves the one sample t-test, the average value of students' tests, analysis-based stages of problem-solving, analysis-based indicators of critical thinking skills in general, and analysis-based indicators of the mathematical critical thinking. Data from in-depth interviews were analyzed qualitatively to verify the results of students' answers based on the aspects of students' critical thinking abilities. The following is an explanation of each analysis.
To test the effectiveness of the application of the mathematics learning model to stimulate critical thinking, it was carried out through a one sample t-test analysis. The analysis was performed by using SPSS 21 for windows software at a significant level of 5%. Before the hypothesis test conducted on the data, the data was tested by the assumption test. The assumption test included normality test. Normality test was not conducted on the data obtained because the sample of research more than 30 students so that the data declared normal distribution. The t-test was carried out on the sample in all research classes at six high schools as many as 141 students. The hypothesis in this study are:

H₀: The mathematics learning model is not effective in stimulating students' critical thinking skills

H₁: The mathematics learning model is effective in stimulating students' critical thinking skills

The decision-making data for one sample t-test are:

1. If the value is Sig. (2-tailed) < 0.05, then H₀ is rejected
2. If the value is Sig. (2-tailed) > 0.05, then H₀ is accepted

In terms of the average test scores analysis, the critical thinking ability test results are converted to the total score of a 1-100 scale. This value determines whether or not the students' grades have reached the minimum mastery criteria (KKM) score of 75. Furthermore, the analysis rests on the problem-solving stages, namely the students' scores are grouped into the eleven stages of problem-solving. The stages of problem-solving questions measure the ability to think critically on the material of geometry transformation, as follows: 1) Writing things and problems based on the questions, 2) Painting geometric shapes based on problems, 3) Painting the transformation lines, 4) Painting the results of transformation, 5) Solving problems using the right steps coherently, 6) Drawing conclusions, 7) Providing explanations concerning the conclusions, 8) The principle of transformation, 9) The principle of substitution, 10) Considering the definition of transformation, 11) Using the substitution procedures to determine the shadow equation.

The next analysis is indicators of critical thinking skills. The students' grades are grouped into seven indicators of critical thinking, namely (1) focusing questions; (2) asking, answering questions, clarifying challenging questions; (3) considering whether or not the source can be trusted; (4) inducing, considering the results of induction; (5) creating and studying the values and the results; (6) defining terms, considering definitions; and (7) determining actions.

The last analysis is based on the mathematics critical thinking indicators to determine the criteria for students' mathematics critical thinking abilities. Eight indicators of mathematics critical thinking have been determined namely identifying questions, giving simple explanations, using correct procedures, careful habits, drawing conclusions, applying principles, considering definitions, and formulating solutions. The criteria for students' critical thinking skills are summed up in the table below.

<table>
<thead>
<tr>
<th>Grade</th>
<th>The Grade Interval</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>80,0 &lt; M ≤ 100,0</td>
<td>Very good</td>
</tr>
<tr>
<td>B</td>
<td>60,0 &lt; M ≤ 80,0</td>
<td>Good</td>
</tr>
<tr>
<td>C</td>
<td>40,0 &lt; M ≤ 60,0</td>
<td>Average</td>
</tr>
<tr>
<td>D</td>
<td>20,0 &lt; M ≤ 40,0</td>
<td>Below average</td>
</tr>
<tr>
<td>E</td>
<td>0,0 ≤ M ≤ 20,0</td>
<td>Very bad</td>
</tr>
</tbody>
</table>

Findings / Results

The effectiveness of applying the mathematics learning model to stimulate critical thinking

The results of the one sample t-test using SPSS can be seen in Table 3 below.

<table>
<thead>
<tr>
<th>One-Sample Test</th>
<th>t</th>
<th>df</th>
<th>Sig. (2-tailed)</th>
<th>Mean Difference</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Score</td>
<td>14,649</td>
<td>140</td>
<td>0.000</td>
<td>9,18156</td>
<td>7,9424 - 10,4207</td>
</tr>
</tbody>
</table>

The one sample t-test result shows that the application of the mathematics learning model is effective in stimulating students' critical thinking skills, \( t(140) = 14.649, p = 0.000 \).
Based on table 3 above, it is found that the significance value $t$ is 0.000 less than 0.05. It means, $H_0$ is rejected, it means that the application of the mathematics learning model is effective in stimulating students' critical thinking skills.

The Test Results Analysis Based on the Students' Grades

The data on the results of students’ critical thinking skills tests in each school can be viewed in the table and figure below.

**Table 4. Results of critical thinking abilities tests based on the students’ grades**

<table>
<thead>
<tr>
<th>No</th>
<th>Schools</th>
<th>Subjects</th>
<th>Average Grades</th>
<th>Standard deviation</th>
<th>Percentage of the Classical Completeness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SMA N 2 Pacitan</td>
<td>28</td>
<td>84.39</td>
<td>8.01</td>
<td>100 %</td>
</tr>
<tr>
<td>2</td>
<td>SMA N Punung</td>
<td>21</td>
<td>85.54</td>
<td>7.92</td>
<td>100 %</td>
</tr>
<tr>
<td>3</td>
<td>SMA N Tulakan</td>
<td>22</td>
<td>83.84</td>
<td>7.72</td>
<td>100 %</td>
</tr>
<tr>
<td>4</td>
<td>SMA N 1 Ngadirojo</td>
<td>24</td>
<td>85.03</td>
<td>6.94</td>
<td>100 %</td>
</tr>
<tr>
<td>5</td>
<td>SMA N 2 Ngadirojo</td>
<td>23</td>
<td>82.77</td>
<td>7.50</td>
<td>100 %</td>
</tr>
<tr>
<td>6</td>
<td>SMA N Tegalombo</td>
<td>23</td>
<td>83.52</td>
<td>6.88</td>
<td>100 %</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td></td>
<td>84.18</td>
<td>7.49</td>
<td>100 %</td>
</tr>
</tbody>
</table>

Table 4 shows that all students’ grades in the six schools reached the minimum mastery criteria. This means that all students have achieved individual completeness, so the percentage of completeness is 100%. The mathematics learning model stimulates critical thinking skills. In other words, it positively influences high school students’ mathematical critical thinking skills.

The Analysis of Critical Thinking Abilities Based on the Test Items Completion Stages

In general, test results are analyzed based on aspects of critical thinking skills. The analysis was carried out to identify scores based on the test items completion stages, as shown in the table below.

**Table 5. Results of critical thinking abilities tests based on the test items completion stages**

<table>
<thead>
<tr>
<th>No</th>
<th>Schools</th>
<th>Test Items Completion Stages</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>SMA N 2 Pacitan</td>
<td>93.2</td>
</tr>
<tr>
<td>2</td>
<td>SMA N Punung</td>
<td>92.4</td>
</tr>
<tr>
<td>3</td>
<td>SMA N Tulakan</td>
<td>95.5</td>
</tr>
<tr>
<td>4</td>
<td>SMA N 1 Ngadirojo</td>
<td>95.0</td>
</tr>
<tr>
<td>5</td>
<td>SMA N 2 Ngadirojo</td>
<td>90.0</td>
</tr>
<tr>
<td>6</td>
<td>SMA N Tegalombo</td>
<td>93.0</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>93.2</td>
</tr>
</tbody>
</table>

The data on the results of students' mathematics critical thinking skills are analyzed based on the test items completion stages as summarized in the table and figure below.
The results of students’ mathematics critical thinking skills are reflected through the test questions. The first stage of problem-solving in the critical thinking ability test is writing things in the perspective of problem-based questions. The analysis of students’ work outcomes show that most students can write things well, and solve problems. Only a few results of the work are incomplete or unwritten using a number of questions. After an in-depth interview, it is known that the reason for not writing this aspect using several numbers results from the students’ worries, and limited time to solve all the questions.

The second stage of the problem-solving is to paint geometric shapes based on the emerging problems. In this sense, students are asked to paint lines, circles or triangles concerning mathematics critical thinking instructions. At this stage, some work results are inappropriate, both in shape and size contexts. Other work results are not neat, because they do not use rulers or terms. However, the overall results are in the good category of competence.

The third stage of the problem-solving is to paint the transformation line. The first question deals with a reflection line, the second problem is a translation line, the third problem is painting a rotation angle, the fourth problem is painting the center of dilation, and the fifth problem is painting the reflection of a triangle. In this aspect, the results of students’ work are almost entirely true. Only a few results of the work are not quite right, nor are they evenly distributed to all questions.

The next stage of the problem-solving is to paint the results of transformation. In this dimension, students are expected to paint the results of transformation in the form of reflection, translation, rotation, dilation, and composition of the two transformations correctly. Test results on this activity are not very satisfactory. The analysis of the test results shows that the average number of questions that got a low score in all schools was on question number 4. For this reason, students considered the question number 4 more difficult than other questions. However, the overall grades are in the good category of achievement.

The fifth stage of the problem-solving is to figure out problems correctly and coherently. This aspect is another step of the problem-solving (an alternative solution) in addition to the drawing method, namely the problem-solving using the matrix method or substitution method. Thus, if students can complete this means of problem-solving, they have thought critically, whereas one problem can be solved by using several steps of completion, but with the same results. The average results of students’ work on this aspect are mostly correct but incoherent resulting in fairly low scores. Nevertheless, the final test results are in the good category of competence.

The sixth and seventh stage of the problem-solving is to draw conclusions and provide explanations concerning the given conclusions. In this activity, students are expected to have the ability to draw conclusions on the completion of the problems while providing an explanation. The analysis shows that the results on both aspects are very good in all schools.

The next stage of the problem-solving is mainly concerned with the principle of transformation consisting of the principles of reflection, translation, rotation, dilation, and the composition of transformation. The results reached the excellent category (100%), and this shows that students have mastered concepts of geometry transformation well.

<table>
<thead>
<tr>
<th>Sub-indicators</th>
<th>Test Items Completion Stages</th>
<th>Average Grades</th>
<th>Grades Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Identifying questions</td>
<td>1. Writing things around the problems based on the questions</td>
<td>93,2</td>
<td>Very good</td>
</tr>
<tr>
<td>2. Simply explaining</td>
<td>2. Painting geometric shapes based on the problems</td>
<td>80,8</td>
<td>Good</td>
</tr>
<tr>
<td>3. Use the procedure</td>
<td>3. Painting the transformation lines</td>
<td>97,6</td>
<td>Very good</td>
</tr>
<tr>
<td>appropriately</td>
<td>4. Painting the results of transformation</td>
<td>76,2</td>
<td>Good</td>
</tr>
<tr>
<td>4. Exercise caution</td>
<td>5. Solve problems with the right steps and coherent</td>
<td>70,4</td>
<td>Good</td>
</tr>
<tr>
<td>5. Make conclusions</td>
<td>6. Make conclusions about the results of the transformation</td>
<td>94,9</td>
<td>Very good</td>
</tr>
<tr>
<td>7. Provide an explanation</td>
<td>7. Provide an explanation of the conclusions written</td>
<td>91,3</td>
<td>Very good</td>
</tr>
<tr>
<td>of the conclusions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Applying the principles</td>
<td>8. The principle of transformation</td>
<td>100</td>
<td>Very good</td>
</tr>
<tr>
<td>9. The principle of substitution</td>
<td></td>
<td>88,7</td>
<td>Very good</td>
</tr>
<tr>
<td>7. Considering the</td>
<td>10. Considering the definition of transformation</td>
<td>100</td>
<td>Very good</td>
</tr>
<tr>
<td>definition</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Determining the action</td>
<td>11. Using the substitution procedure to determine the shadow equation</td>
<td>72,6</td>
<td>Good</td>
</tr>
</tbody>
</table>

Table 6. Results of critical thinking abilities tests
The principle of substitution is the completion of the ninth question. In this part, students are required to solve the problems beyond the drawing method commonly called the substitution method. The analysis of test results is in the excellent category as seen from the problem-solving stage that requires the principle of substitution, and students can solve the problems. This is because the solution to the substitution method was studied in the previous chapter. Thus, students can solve problems in several ways.

The tenth stage of the problem-solving is to consider the definition of transformation. This issue considers the definition of reflection, translation, rotation, and dilation. To solve the problems, students use both drawing and substitution methods, and students are expected to use the definition of transformation. The analysis of the students’ work shows 100% results that have met the criteria, as can be viewed from the completion of all given problems.

The final stage of the problem-solving is to use the substitution procedure to determine the shadow equation. The previous aspect has been discussed regarding the principle of substitution. In this respect, the principle of substitution is used to determine the shadow equation. The analysis of scores shows quite good results, but is still lower than other aspects because students are not careful enough in solving problems using the substitution method resulting in the less precise scores.

In general, the average scores in each stage of completion of the critical thinking skills test questions for all students from six schools have reached the good category of competence. This indicates that students have good critical thinking skills after applying the mathematics learning model to stimulate critical thinking skills.

The Analysis of Critical Thinking Abilities in terms of General Critical Thinking Indicators

The analysis of problem-solving stages is narrowed down to the data analysis related to indicators of critical thinking abilities. The results of the analysis for each indicator are outlined in the following table and figure.

*Table 7. The analysis of test results based on critical thinking indicators*

<table>
<thead>
<tr>
<th>Indicators</th>
<th>Average Grades</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focusing on the question</td>
<td>93,2</td>
<td>Very good</td>
</tr>
<tr>
<td>Asking questions, answering questions, clarifying challenging questions</td>
<td>89,2</td>
<td>Very good</td>
</tr>
<tr>
<td>Considering whether or not the source can be trusted</td>
<td>73,3</td>
<td>Good</td>
</tr>
<tr>
<td>Inducing, considering the results of the induction</td>
<td>93,1</td>
<td>Very good</td>
</tr>
<tr>
<td>Making and reviewing the values of the results worth considering</td>
<td>94,3</td>
<td>Very good</td>
</tr>
<tr>
<td>Defining terms, considering definitions</td>
<td>100</td>
<td>Very good</td>
</tr>
<tr>
<td>Determining the action</td>
<td>72,6</td>
<td>Good</td>
</tr>
</tbody>
</table>

The information from Table 7 concerning the largest average grade is the indicator that defines the term and considers the definition, while the indicator determines which actions have the lowest grade.

Focused questions are an integral part of the focused aspect. Ennis (Ennis, 1993) states that focused questions are the first step to make a decision concerning what to believe or what to do. In general, these indicators include sub-indicators of identifying or formulating questions, identifying or formulating criteria to consider possible answers, and maintaining conditions of thinking (Setiana et al., 2019). The sub-indicators of mathematics critical thinking are identifying questions and maintaining conditions of thinking. This indicator is closely connected to general indicators on the aspects of abilities, (Ennis, 1993) namely prioritizing a specific issue, focusing on the main problems, and questions immediately clarified.

Asking questions, answering questions, and clarifying challenging questions mean that the teacher asks questions using learning media, challenges students, and assigns them to make questions, and answer their friends’ questions. This indicator is part of the focused aspect (Ennis, 1993). In general, these indicators include sub-indicators that provide simple explanations and cite examples. The sub-indicators of mathematics critical thinking seeks to provide simple explanations and cite examples. This indicator connected to the general indicators (Zubaidah et al., 2015) in the aspect of abilities, namely asking questions to understand things in the aspect of dispositions, identifying goals and planning before answering questions.

To consider whether or not the source can be trusted in the framework of critical thinking is part of the reason that the activity determines the cause of the emergence of a thing by identifying and evaluating a reason. Ennis et al., 2015) states that relevant information can provide a reason for making a decision. The conclusion is that the teacher has not developed indicators of critical thinking skills that can help students determine a major problem and check the relevance of information to solve the problem at hand. In general, these indicators have sub-indicators by considering expertise, conflict attractiveness, source appropriateness, reputation, appropriate procedures, risks to reputation, ability to reasons, and cautious habits (Setiana et al., 2019). The sub-indicators of mathematics critical thinking consider the appropriateness of the source, the use of appropriate procedures, the ability to give reasons, and careful
habits. This indicator relates to the general indicator (Ennis, 1996) in terms of the aspect of abilities, which is to consider models or opinions and related prior knowledge to the new one, and to aspects of dispositions, which emphasize the identification of information before starting to answer questions and searching for information needed. Besides, general indicators contain the indicator content on the aspect of problem-solving, namely finding out contextual information, the linkages and deviations of information.

To consider the results of an induction means to investigate, collect data, make generalizations from the existing data, make tables and figures, and draw hypothetical conclusions. The term inductive refers to the specific-to-general conclusions (Sulistyowati et al., 2019). Induction is an integral part of the inferences aspect in the critical thinking framework (Zubaidah et al., 2015) which is an activity to show further information concerning a matter. In general, these indicators include sub-indicators that put forward general things, conclusions and hypotheses, design experiments, conclusions related to facts, and conclusions from the results of investigations. The sub-indicators of mathematics critical thinking seek to express general things, present conclusions and hypotheses, and draw conclusions from the results of investigations (Setiana et al., 2019). This indicator closely connects to the general indicators and aspects of abilities, namely using statements and symbols appropriately to provide information in a systematic way, and in a logical order. In addition to the aspect of ability, the aspect of tendencies (dispositions) include showing information through graphics, pictures, and so forth. Besides, general indicators contain the problem-solving aspect, namely finding a common form of the target of resolution, choosing and justifying a strategy to solve the problems, finding and concluding the goals that lead to the goal.

Reviewing the results of grades means providing an understanding of materials and activities. This indicator is part of the inferences aspect of the critical thinking framework (Ennis, 1993). In general, these indicators include sub-indicators, make and determine the results based on the background facts, make and determine the results based on the consequences, make and determine the results based on the application of facts, and make and determine the results of balance and problem considerations (Purwoko et al., 2019). The sub-indicators of mathematics critical thinking are making and determining the results based on the background facts and making and determining the results of problem-solving including applying mathematics principles. The indicators of critical thinking (Ennis, 1993) focus on the indicator content of generalization aspects that include finding concepts, finding conditions for applying concepts, finding formulas that are different from generalizations (special situations), and showing evidence to support generalizations.

Defining terms and considering definitions mean giving students opportunities to mention simple definitions of terms through questioning activities. This indicator is categorized as situation and clarification aspects. In general, these indicators have sub-indicators such as making the form of definitions, defining strategies, and acting, providing further explanation, identifying and dealing with intentional untruthfulness, and making the contents of the definitions (Setiana et al., 2019). The sub-indicators of mathematics critical thinking are likely to form definitions, consider definitions and provide further explanation (Setiana et al., 2019). This indicator is closely connected to the content indicator (Ennis, 1996) through the concept-related aspect, namely identifying the characteristics of concepts, comparing concepts with other concepts, identifying examples of concepts with justification and identifying concepts followed by giving the reason, besides the concept aspect on algorithms and skills that explain the basic concepts of skills.

Determining the action means deciding and implementing that includes choosing a possible solution and determining the likelihood of the action carried out through discussions, exercises, giving directions and questions. To determine the actions means to include the overview aspect. In general, these indicators have sub-indicators, namely revealing problems, choosing criteria to consider possible solutions, formulating alternative solutions, determining interim actions, repeating, and observing their application (Setiana et al., 2019). The sub-indicators of mathematics critical thinking are likely to choose the criteria by considering possible solutions and alternative solutions (Setiana et al., 2019). This indicator deals with the general indicator (Ennis, 1993) on the dispositions aspect, namely testing the solution. In addition, general indicators have the indicator content about the problem-solving aspect such as creating a form common to the target of resolution, finding out the information in context, linkages and deviations of information, choosing and justifying a strategy to solve problems, finding and concluding goals that lead to goals, propose alternative methods to solve problems and find similarities and differences among given problems.

The Analysis of Mathematics Critical Thinking Abilities

The analysis of mathematics critical thinking skills results in some indicators of mathematics critical thinking as viewed in the following table and figure.
Table 8. The tests result of mathematics critical thinking skills

<table>
<thead>
<tr>
<th>Sub-indicators</th>
<th>Grades</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Identifying questions</td>
<td>93.2</td>
<td>Very good</td>
</tr>
<tr>
<td>2. Simply explaining</td>
<td>89.2</td>
<td>Very good</td>
</tr>
<tr>
<td>3. Using the procedure appropriately</td>
<td>76.2</td>
<td>Good</td>
</tr>
<tr>
<td>4. Exercising the caution</td>
<td>70.4</td>
<td>Good</td>
</tr>
<tr>
<td>5. Drawing conclusions</td>
<td>93.1</td>
<td>Very good</td>
</tr>
<tr>
<td>6. Applying the principles</td>
<td>94.3</td>
<td>Very good</td>
</tr>
<tr>
<td>7. Considering the definition</td>
<td>100</td>
<td>Very good</td>
</tr>
<tr>
<td>8. Formulating a solution</td>
<td>72.6</td>
<td>Very good</td>
</tr>
</tbody>
</table>

Table 8 shows the indicator of the highest grade in terms of critical thinking skills is the indicator to consider the definitions, while the lowest grade is the indicator of caution habits.

The indicator of identifying questions is regarded as a very good category. Students learn to think critically through practiced habits in the form of formulating problems and answering questions that require explanations (Leicester & Taylor, 2010). In this indicator, students are asked to write things known as problems related to the questions on the answer sheets provided. Using pictures, students can understand the problem-solving aspect through the information provided.

The Results of In-depth Interviews with Students

The results of in-depth interviews with students are presented in Table 9 below.

Table 9. The Results of In-depth Interviews

<table>
<thead>
<tr>
<th>Aspect</th>
<th>Interview result</th>
</tr>
</thead>
<tbody>
<tr>
<td>F (Focus)</td>
<td>Students are able to convey what information is on the questions and mention the questions that are meant in the questions</td>
</tr>
<tr>
<td>R (Reason)</td>
<td>Students are able to explain the relevant reasons at each problem solving step written on the answer sheet</td>
</tr>
<tr>
<td>I (Inference)</td>
<td>Students are able to provide reasons used to formulate conclusions, reasons are sufficient to support the conclusions made, relevant, sufficiently detailed, and clear Some students are not accustomed to writing conclusions every time they solve a problem, but students are able to give conclusions orally.</td>
</tr>
<tr>
<td>S (Situation)</td>
<td>Students are able to distinguish important and needed information in solving problems and information that is less important</td>
</tr>
<tr>
<td>C (Clarify)</td>
<td>Students are able to explain back what they wrote at the final conclusion clearly and smoothly. Students are able to explain in detail related to the terms asked by the researcher contained in the questions. Students are able to provide examples of similar cases with the questions given orally.</td>
</tr>
<tr>
<td>O (Overview)</td>
<td>Students also try to examine work results by linking to the context of the problem that has been solved. By checking the results of work, students have confidence that the answers that have been obtained are correct</td>
</tr>
</tbody>
</table>

Based on Table 9, it is known that the results of in-depth interviews generally show results that show and tend to strengthen the test results, indicating that the students have fulfilled the critical thinking aspects. Some things that students did not do in writing could be recorded through in-depth interviews.

Discussion

Broadly speaking, the learning process refers to the scientific learning approach concerning the 2013 curriculum. The sheer focus is on the mathematics learning models that stimulate critical thinking skills through the core activities and learning tools. Mathematics learning models stimulate the critical thinking skills in the four consecutive meetings for each research class. After implementing this learning model, tests are measured to evaluate the students’ critical thinking skills. Test questions are based on the test grid that refers to indicators of critical thinking skills. The test grid is prepared by reviewing the indicators and sub-indicators of critical thinking skills as an integral part of the test items completion stages.

Based on the analysis of the previous findings some indicators provide a simple explanation categorized as an excellent achievement. Simple explanations indicate that the students' abilities explain how the cause and effect relation works in solving problems. In this indicator, students can explain to what extent the arguments, statements or opinions are conveyed so that they are arranged in a strong organization (Aslan & Aybek, 2020; Agoestanto et al., 2019;
Providing an explanation is the students’ ability to express the results of data, evidence, opinions, or questions. Students commonly use this indicator to identify the results of the acquisitions of grades in a very good category (Salbiah, 2017). The problem-solving ability in this indicator was improved through practice, observation and discussion. Several activities that provide explanations can train students to develop the students’ reasoning and critical thinking skills (Rodzalan, et al., 2015; Utami, et al., 2019; Pujiyono, 2012). In this indicator, students are asked to draw geometrical objects such as circles, triangles, squares, or lines and determine the type of transformation ordered in the problem-solving component while describing the intended transformation lines. In this case, most students have the abilities to describe geometry objects as objects that will be subject to the principles of transformation. Students can also distinguish the types of transformations and understand the concept of transformation embedded in the problems.

Indicators of setting procedures correctly are included in either category. This relates to the students’ abilities to interpret and describe problems (Ghazivakili, et al., 2014). Using procedures appropriately trains students to re-explain and understand the meaning of a concept, principle, or procedure that has been learned previously. Students provide feedback and re-express the procedure through the completion of the problem. An important part of the students’ critical thinking is to learn in a systematic way of thinking and re-explaining (Su et al., 2016; Hakim, et al., 2018). In this indicator, students are asked to solve problems in the first way using the drawing method. Students are expected to paint the results of the objects transformation that have been previously undertaken. Transformation is in the form of translation, reflection, rotation or dilation. Almost all students have determined the transformation results as well. The problem number 4 contains the concept of dilatation that many students cannot complete when solving problems by way of the drawing method. Even some students do not show their competence so that the acquisition of the average grade is lower than the other indicators.

Indicators of caution habits are included in either category. In this indicator, students are asked to solve problems correctly and coherently. In each question, there is a question “Is there another way to solve the problem?” If yes, write it down completely and coherently. The question type can students develop critical thinking skills through answering innovative questions (Thinking, 2015). These types of questions are open-ended, whereas each of them is worked out in several ways. Potts (1994) conclude that some of the characteristics of critical thinking learning include asking open-ended questions. Hidayati (2017) also mention one of the characteristics of teaching to think critically, including asking open-ended questions. Romberg (1995) points out that to build critical thinking in the students’ learning environment is to face the contradictory and new problems so that students construct ideas in search of truth and clear reasons. To get a score on this indicator, students must solve problems using the drawing method and other methods, such as substitution or matrix. By solving the problem with the right steps and coherent means, students have met the indicators of mathematics critical thinking commonly called the habit of caution. Most students have solved the questions correctly but in a less coherent order so that the achievement of this indicator is at the lowest score in either category.

The indicators of drawing conclusions include the excellent category. To draw conclusions means to show the relationship between a number of ideas, draw the right conclusions, generalize, explain (not describe) and make hypotheses (Perkins & Murphy, 2006). The fulfillment of this indicator includes drawing conclusions correctly by providing an explanation of the conclusions. In this indicator, students are expected to draw reasonable conclusions (Sasi, 2018). An important part of this inference step is to identify assumptions and seek solutions, interpret the situation and evidence. Students are likely to identify and solve a problem in drawing a conclusion. Conclusions are drawn to facilitate what has happened or been observed (Purwoko, 2017). One fulfillment of this indicator is that students are asked to draw conclusions correctly. The results of the research show that most students can write conclusions correctly even though only a single conclusion was obtained from several methods of completion. The category of achievement indicators is very good if students have the ability to think critically. Onions (Bamford, 2012) argues that critical thinking results in valid arguments and conclusions, and strengthens the resistance to criticism. Other opinions that support the ability to think critically can also be improved by giving investigative questions, encouraging students to solve problems and draw conclusions based on the inquiry (Iakovos, 2011).

Providing an explanation of conclusions is the second condition on the fulfillment of indicators of drawing conclusions. Therefore, it is reasonable to support or oppose decisions made concerning relevant situations and facts. To formulate arguments means to support conclusions, look for evidence that elaborates the reasons for an acceptable conclusion so that students can identify and justify the problem. The likelihood of well-organized arguments deals with how students solve the main problem, make decisions, consider all possible aspects, study and conclude them logically. This is done not only at the end, but done as long as students solve the problem (Zubaidah et al., 2015).

Not all information is always true or relevant, therefore, it is necessary to identify some criteria such as clarity, accuracy, accuracy, reliability, and other evidence that supports the argument in drawing conclusions. To draw a correct conclusion, information is required to identify mathematics properties relevant to the problem-solving at hand. If the decision is not based on the correct information and assumptions, then the conclusion has no correct basis. As Jennifer (Mulnix, 2012) argues, critical thinking does not directly lead to conclusions or acceptance of some evidence, demands or decisions without really thinking about them, but critical thinking clearly requires interpretation and
evaluation of observations, communication and other sources of information. Lambertus (2009) adds that mathematics materials and critical thinking skills are two inseparable things because mathematics materials are understood through critical thinking, and critical thinking is trained through the pedagogy of mathematics. Thus, working on mathematics cannot immediately get an answer; instead there must be evidence that supports students' arguments.

Indicators of applying principles include an excellent category. The principle pertains to the relationship between some basic mathematics objects so that it consists of several facts, concepts and is associated with an operation (Amir, 2015). These principles include the equation of the circle, the principle of transformation, both through the method of drawing and alternative methods using the principle of substitution and matrix. In this indicator, students are asked to apply acceptable principles. The achievement of the excellent category indicates that students can apply the principles learned through the material transformation of geometry and the completion of problems while they have the ability to think critically in terms of indicators of critical thinking.

The indicator considers that the definition is an excellent category. Even the overall average score reaches a perfect score of 100. A concept is learned through definition. A definition is an expression that limits concepts. Through the definition, people can describe, or illustrate, or make a scheme, or make a symbol of the concept (Amir, 2015). The learning process with a mathematics learning model stimulates the critical thinking that leads students to build their own knowledge. Through the stages of building basic skills, they learn materials including definitions and concepts. Many exercises given at the time of learning help students understand definitions and learn solve problems by providing further explanations.

Indicators of formulating solutions are stated in each category. In completing a number of questions, students are required to associate several concepts to formulate a solution until the final answer is obtained. One of the objectives of mathematics subjects is to develop students' abilities in understanding mathematical concepts, explaining the interconceptual relationships and applying concepts and algorithms flexibly, accurately, efficiently, and precisely when solving problems (Permendiknas, 2005) concerning Competency Standards and Basic Competencies. Chen (2015) states that the ability to learn mathematics is interrelated. The statements in mathematics are obtained through a deductive mindset, meaning that the truth of a statement in mathematics must be based on previous mathematical statements recognized as true. Besides, the system in mathematics adheres to the law of consistency as the structure of mathematics materials is arranged very hierarchically and interrelated. As a result, learning mathematics occurs when the mastery of a material directly influences the ability to learn further materials. In terms of formulating solutions, students can relate the concepts of geometrical transformation such as translation, reflection, rotation, and dilation with the principle of substitution to obtain the correct final answer.

Regarding interviews, in-depth interviews aim to dig up information that cannot be found through tests. In the "focus" aspect, students are able to explain the information given to the questions containing geometric transformation by using oral mathematical sentences. Some students were able to provide information using their own sentences, but some students still presented information by reading questions, however, both of them still show their ability to fulfill the focus aspect.

In the aspect of "reason" students are retained the answer sheets of each work complete with the steps to solve the problem. Furthermore, students are asked to explain their reasons for writing these steps. Most students can explain the reasons coherently to conclusions, such as the use of elimination or substitution methods to solve the transformed image equation.

The 'inference' aspect gives students the opportunity to convey the reasons for the conclusions that have been written. In this case all students can explain well. In the 'situation' aspect, the interview questions focus on what important information is needed in solving the problem and are able to define terms related to the material being studied. In this case students are able to define translation, rotation, reflection and dilation. In the 'clarify aspect, students are required to be able to provide examples of questions that are similar to the questions given. Some students can compile similar questions in their own language and present new problems but still in context, while other students only replace numbers on the same question.

In the aspect of "overview" the researcher digs up information related to checking the results of problem solving again. In this case most of the students reviewed the work results from beginning to end, but there were also students who only researched at the end. By examining the results of the work, it can be seen whether the problem solving is correct or something needs to be fixed.

Conclusions

After applying the mathematics learning model that stimulates critical thinking, students show their useful mathematics critical thinking skills as described through (1) The results of the one sample t-test show that the application of the mathematics learning model is effective to stimulate critical thinking; (2) Overall students’ grades that meet the minimum mastery criteria (KKM); (3) The results of the analysis based on the eleven stages of the problem-solving, and the criteria of students' critical thinking abilities are categorized as good and excellent, along with the stages in which the highest score represents the principle of transformation and the definition of transformation.
while the lowest grade is recognized from the stage of the problem-solving using right and coherent steps; (4) the results of the analysis is based on the indicators of critical thinking skills of seven indicators that show the criteria of students’ critical thinking abilities in the category of good and very good scores with indicators of the highest grade in defining the term and considering the definition, while the indicator for the lowest grade is a determining action; (5) the results of the analysis is based on the indicators of mathematics critical thinking of eight indicators that prove that the criteria of students’ critical thinking abilities are in the good and very good categories with indicators of the highest grade by considering the definition, and the indicator of the lowest grade is the habit of caution.

**Recommendations**

The results showed that the application of the mathematics learning model to stimulate critical thinking which was analyzed based on one sample t-test analysis, analysis of students’ total scores, analysis of problem solving stages, analysis based on indicators of critical thinking skills, and analysis based on indicators of mathematical critical thinking skills as a whole showed positive results on students’ critical thinking skills. Thus, this learning model can be used as an alternative in choosing a mathematics learning model to improve mathematical critical thinking skills, especially at the junior high school level. For further research, it is expected to carry out more in-depth analysis related to indicators of mathematical critical thinking.

**Limitations**

Research on the application of mathematics learning models for people who think critically in senior high school students, problem analysis, analysis based on indicators of mathematical critical thinking skills, and learning materials on geometric transformation.

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