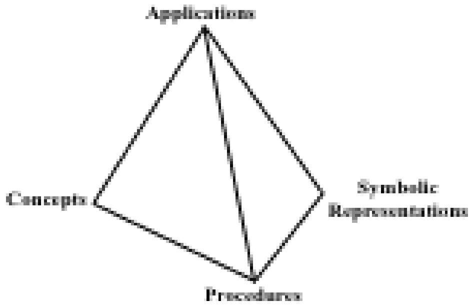

Figure 1. The Mathematics Tetrahedron

The work of Piaget (1963) has established that the young learner is trying to make sense of what is experienced. In mathematics, trying to master the processes and symbolism may well create enough pressure on limited working memory capacity. The learner cannot cope with concepts (understandings), procedures, symbolisms and applications all *at the same time*.

Adding on the '*making sense of*' dimension will almost certainly generate overload and yet it is this dimension which is the natural way of learning. Ausubel (1968) talks of meaningful learning where what is understood is '*internalised*'. This is critical but the limitations of working memory may make such understanding and internalisation very difficult. There are good arguments, therefore, for making the mathematics taught meaningful. There are also good arguments that this may prove very difficult.

The key has to be in establishing confidence and competence in the processes and symbolisms at one point in time and, later, adding on the understanding. In this way, the limitations imposed by working memory may be reduced. With the procedures more or less automated, enough working memory space is now available to think about their meaning and application.

Development of Attitudes towards Mathematics

Learners have often perceived mathematics as a difficult subject (Brown *et al.* 2008). Mathematics is abstract and logical in nature and many concepts in mathematics cannot be explained easily in terms of physical representations or related to every day life. It is possible that mathematics is able to contribute to the all-round development of each student by offering understanding which will enable them to develop socially in the sense of being able contribute to society more effectively and being able to make sense of society more effectively. However, mathematics is often portrayed as being abstract and unrelated to life.

In many countries, many students do not enjoy school mathematics and seek to avoid it later (McLeod, 1994; Brown *et al*, 2008). Attitudes tend to deteriorate with age simply because work becomes more demanding and also because, as the students get older, they may start to think that they will not need mathematics in the future. Indeed, negative attitudes towards mathematics are difficult to undo (Duffin & Simpson, 2000).

Al-Ahmadi and Oraif (2009) looked at the idea of academic confidence and found it related almost exclusively to past success. This has enormous implications for mathematics. If the students see success as elusive, then it is not likely that attitudes will remain positive. If confidence is mostly related to examination success only and is not dependent on other factors, then examinations are almost doomed to consign certain students to the dustbin of failure and their confidence will be damaged.

While the demand of challenging tasks does not, of itself, necessarily generate a perception of difficulty, when a task is perceived as being so difficult that the effort is not justified by the rewards, attitudes seem to deteriorate (Reid & Yang, 2002; Reid & Skryabina, 2002). Alhmali (2007) found that the perceptions of Libyan school students (ages 12-18) were that mathematics was not an easy subject, it was over-abstract, and the least enjoyable and least attractive part of school. In his results, '*polarisation*' of views was quite remarkable: while some students '*loved'* it, many students '*hated'* it.

In a recent study, Jung and Reid (2009) found a relationship between attitudes towards the sciences and measured working memory capacity. Of even greater significance, they found that those with *lower* working memory capacities tended to rely *more* on memorisation while those with higher working memories were more reliant on understanding. However, both groups appeared to *want* to understand. Perhaps this offers the key: negative attitudes relate to *failure to understand*. The natural process is to seek to make sense of things. When this is impossible or the opportunities are denied them, then attitude problems will arise.

Teachers make a huge impact on learner attitudes (Ponte *et al.,* 1991; Johnstone and Reid, 1981; Reid and Skrybina, 2002) and there is a remarkable relationship between the views and attitudes of the teachers and those of their students (Al-Enezi, 2004). Brown *et al.* (2008), working with a sample of over 1500 students in 17 schools in England, investigated student attitudes towards mathematics. These students were close to the moment of curriculum choice and the results showed that the main reason for opting out was perceived difficulty. However, the learners may be receiving external messages about difficulty and these may suggest that their future in mathematics may be a struggle. This may accelerate the feeling of being unable to understand mathematics (Matrhews & Pepper, 2005; Kyriacou and Goulding, 2006). Ma and Kishore (1997) have brought many of the findings together by considering how all these influences generate identities relating to mathematics: *'low expectations create low attainment as much as they respond to it'*.

Matthews and Pepper (2005) note powerful reasons for not continuing with mathematics. These include lack of enjoyment and a belief that the subject is boring, for both high attaining as well as low attaining students. They also saw the perceived irrelevance to the real world as a factor. Indeed, at age 11-14, fun is a key aspect for students of effective mathematics learning (Nardi & Steward, 2003).In England, a predominant reason for taking mathematics is to become a mathematician or to choose a career heavily dependent on mathematics while, in Scotland, the reason for taking mathematics at the upper end of secondary is that mathematics is seen as an integral part of overall education. Mathematics is very popular in Scotland at all levels, the reverse of England.

Experimental

The aim was to explore aspects of mathematics learning experiences in grades 5 to 7 (ages approximately 10-12) in both Urdu medium and English medium schools in Pakistan. This involved looking at student perceptions of their experiences, the nature of the difficulties they have with mathematics and possible reasons for these difficulties. To achieve this, a short survey was used with students in grades 5, 6 and 7, drawn for both language systems. This survey not only looked at the student perceptions related to their experiences in mathematics but also surveyed topic areas to see where they were having difficulties. For grade 5, working memory capacity was measured and their mathematics marks were gained.

Samples Used

Samples of students were drawn from a range of Urdu medium schools in the Lahore area of the country, reflecting a diversity of areas and social backgrounds. For English medium schools, two boys schools were selected.

Table 1. Samples Chosen

Grade	Urdu	Medium	English Medium	Totals
	Girls	Boys	Boys	
5	74	76	150	300
6	33	115	106	254
7	107	51	101	259
Totals	214	242	375	813

A list of all the aspects of mathematics was drawn up and questions were developed to explore these. Lists of topics in the mathematics curriculum for each year group for each language system were drawn up. The draft survey was considered by experienced mathematics teachers and minor amendments incorporated. The survey was translated into Urdu and the translation checked. The survey for grade 5, English medium, is shown in the appendix and used several question formats (Likert, 1932; Osgood *et al.,* 1957; Reid, 2003). The other surveys were identical except for the list of topics covered in question 9.

Measurement of Working Memory Capacity

To determine an individual's working memory capacity, the Figural Intersection Test, developed by Pascual-Leone (1970), was used. There are two sets of simple geometric shapes. The presentation set consists of a number of shapes separated from each other. The test set consists of the same shapes but overlapping, so that there exists a common area which is inside all the shapes of the presentation set. Altogether, there are 20 items in the test. The number of shapes varies from 2 to 8. The test is timed. Every item has to be completed in about 20-25 seconds. Its validity and reliability have been demonstrated by Johnstone and Elbanna (1986, 1989).

The Effect of Working Memory

The working memory capacity of the grade 5 students was measured using the figural intersection test. The descriptive data are shown in Table 2.

Table 2. Working Memory Data

Sample	Minimum	Maximum	Mean	Standard Deviation
300	2	8	4.1	1.2

Grade 5 students have an average age of about 10-11 years. Students of age 10 might be expected to have a mean working memory capacity of 4 and the results obtained here are consistent with this.

The working memory capacity measurements were correlated with the marks in mathematics (standard school examinations) using Pearson correlation. The results are shown in Table 3.

Table 3. Working Memory Correlations

	Sample	Pearson r	Probability
Grade 5 Urdu Medium	150	0.69	< 0.001
Grade 5 English Medium	150	0.43	< 0.001

Working memory capacity measurements will only correlate with performance measurements if one or both of the following conditions are fulfilled (Reid, 2009):

"(a) The teaching and learning process is such that students with higher working memory capacities have an advantage.

(b) The assessment is such that students with higher working memory capacities have an advantage."

Typically, correlation values which lie between 0.2 and 0.65 are obtained when working memory capacities are correlated with examination performance (Reid, 2009). A value of 0.69 is unusually high indicating that about 50% of the variance in their mathematics marks is *controlled* by their working memory capacity. While correlation does not of itself imply causation, the work of Johnstone and El-banna (1986, 1989) shows clearly that causation is involved. The effect of working memory capacity on mathematics performance is illustrated in Table 4.

Table 4. Differences in Mean Marks Related to Working Memory Capacity

Working Memory Capacity	Urdu Medium N = 150	English Medium N = 150
	Mean Mathematics Marks (%)	
Above average	63.0	55.7
Average	50.0	48.1
Below average	39.7	46.4

It is clear that working memory capacity can make a very large difference in examination marks. Al-Ahmadi and Oraif (2009) used factor analysis to show that, while working memory capacity correlates with both recall and understanding, it is a very different factor. In other words, the test performance may be influenced strongly by working memory capacity and yet not reflect ability to remember or understand. Thus, students are being rewarded on the basis of a genetic feature (working memory capacity) which does not necessarily reflect *ability* in mathematics.

Survey Data Analysis

The survey offered a vast amount of data. The aim here is to discuss a little of the data where the information may be applicable well beyond the specific educational structure in Pakistan. To simplify the analysis here, it was noted that quite a number of the items in four of the questions [questions 1, 2, 3 and 5 (24 items in all)] how quite high intercorrelations (Kendall's Tau-b) with each other. The items in these four questions were never designed with any underlying structure in mind. Nonetheless, a factor analysis can offer insights to see if there is any underlying structure to explain these correlations. The data for the entire sample (N = 813) were considered using principal components analysis, with varimax rotation. 9 components accounted for 68% of the variance. Table 5 shows the items which had the highest loading on each of these factors (components), with the likely nature of these nine components.

Table 5. Loadings Table (Highest loadings only)

	Factors (Components)								
	1	2	3	4	5	6	7	8	9
Question	Enjoyment, interest	Easiness, difficulty	Effort	Memorising or understanding	Useful in daily life	Type of revision	Visual-spatial	Extent of help at home	Confidence to understand
1 (part 2)					0.85				
1 (part 3)		0.80							
1 (part 5)	0.80								
2 (part 4)				0.83					
2 (part 6)			0.72						
3 (part 4)						0.82			
3 (part 5)								0.88	
5 (part 2)							0.89		
5 (part 3)									0.73

The identification of these components suggest the factors which may be critically important in influencing the development of positive (or negative) attitudes in relation to the learning of mathematics at school level at these ages. For simplicity, the student responses to these nine questions (which loaded most highly onto the nine factors) will be discussed and the data for the other questions left.

Table 5 reveals that enjoyment and interest are very important in developing positive attitudes related to mathematics. It also suggests that a mathematics curriculum must be of appropriate difficulty: excessive difficulty may have powerful negative effects on attitudes. Two factors are particularly interesting: the memorising-understanding dimension and the perception of usefulness in daily life.

It has shown very clearly that understanding is what learners are seeking but, if the difficulties are too great, the learners have to resort to memorisation with concomitant attitude deterioration (Jung and Reid, 2009). Examinations may have a very powerful effect here. The other factor of interest is the perception or otherwise of the usefulness of mathematics in daily life. This has been found to be critical in the sciences (Reid and Skryabina, 2002) and it seems important here also. It is relatively easy to introduce applications into syllabuses in biology, chemistry and physics at school level. It may be very much more demanding to achieve this in mathematics in that introducing another layer of thought will almost certainly overwhelm the working memory. This issue has been discussed by Al-Enezi (2008) and no easy solution is yet apparent.

Patterns of Responses and Age

The patterns of responses for the nine items are shown for Urdu medium schools for each of the three age groups. Data are shown as percentages for clarity but all statistical calculations use raw data. The response patterns are compared using chi-square as a contingency test (table 6).

Table 6. Patterns for Response from Urdu Medium Schools

N (grade 5) = 150, N (grade 6) = 148, N (grade 7) = 158			**Responses (%)**							χ^2
Usefulness in life	Useful in daily life	***5***	79	17	1	0	1	2	Useless in daily life	65.1 (2) $p < 0.001$
		6	43	16	10	13	16	3		
		7	81	13	4	1	1	1		
Easy of under-standing	Easy to understand	***5***	47	24	15	0	3	11	Difficult to understand	61.6 (6) $p < 0.001$
		6	30	18	12	16	10	15		
		7	65	10	4	2	4	16		
Enjoyment	I want to learn it because I enjoy it	***5***	63	15	1	1	4	17	I do not want to learn it but it is a compulsory subject	24.6 (2) $p < 0.001$
		6	40	11	12	14	16	8		
		7	63	6	8	4	2	17		
Understanding-memorising	I actually understand the procedures in class	***5***	68	2	13	5	3	9	I just memorise the proce-dures in class	26.0 (4) $p < 0.001$
		6	30	14	19	13	11	14		
		7	67	10	4	2	5	11		
Effort	I revise my lessons regular-ly	***5***	64	7	3	1	5	21	I revise them just before the exam or test	45.3 (6) $p < 0.001$
		6	43	15	9	11	10	13		
		7	69	6	4	3	3	16		
Type of revi-sion	Enough revision at school to help	***5***	48	9	11	12	14	5	Not enough revision at school to help to help me understand well	49.4 (10) $p < 0.001$
		6	34	18	14	17	9	9		
		7	63	8	6	4	5	14		
Extent of home help	I understand at school with little extra help from home	***5***	49	9	5	9	10	18	I understand at school with extra help from home	35.0 (6) $p < 0.001$
		6	34	19	7	12	11	18		
		7	61	7	3	1	8	21		
Visual-spatial	Diagram and pictures help me to understand mathe-	***5***	67	26	3	1	3			42.7
		6	38	29	21	11	1			

N $_{(grade\ 5)}$ = 150, N $_{(grade\ 6)}$ = 148, N $_{(grade\ 7)}$ = 158			**Responses (%)**					χ^2
	matics well							(6)
		7	50	34	8	6	3	p < 0.001
Confidence to understand	I can understand the main points easily	*5*	65	21	11	3	1	37.8 (6) p < 0.001
		6	43	23	15	16	3	
		7	57	28	11	4	1	

Three features stand out in the data. Firstly, grade 6 is responding in a very different way when compared to grades 5 and 7. As this sample is drawn from the same schools with the same teachers and the same resources and environment, the differences must be caused by the national curriculum in grade 6. This is simply too demanding. Secondly, there is some polarisation of views in many of the items: a significant proportion are opting for the two extreme positions. This is consistent with what Alhmali (2007) found in Libya with older school students. Thirdly, for the majority, views are very positive. This is encouraging. The patterns of responses for the nine items are shown for English medium schools for each of the three age groups. The response patterns are compared using chi-square as a contingency test (table 7).

Responses tend to be positive and there are very few differences in the response patterns for the three age groups. Thus, it appears that the curriculum difficulties for grade 6 observed for Urdu medium schools are not repeated here, suggesting a more appropriate curriculum. However, there is some evidence of polarisation of views, suggesting a small minority who are negatively disposed towards mathematics.

Areas of Difficulty

The areas which showed the highest indication of difficulty for Urdu medium students are shown in table 10. In grade 5, many areas of perceived difficulty relate to geometry. In grade 6, six topics stand out. However, when comparing the response patterns of grade 6 to those of grades 5 and 7, more topics tended to be rated as difficult and the proportion of students seeing difficulty is very much higher. This confirms that that there is a major problem with the curriculum in year 6.

In grade 7, geometry features quite a bit in areas regarded as '*difficult'* but the greatest problems lie in areas where mathematics is being applied to life: tax, inheritance, business. There may be two problems here Firstly, at this age, there will be a great lack of experience of such areas of life. However, secondly, teaching in these areas requires a mastery of the mathematics involved *at the same time* as its application in life. This may simply reflect overload of limited working memory capacity.

Table 7. Patterns for Response from English Medium Schools

$N_{(grade\ 5)} = 150$, $N_{(grade\ 6)} = 106$, $N_{(grade\ 7)} = 101$		**Grade**	**Responses (%)**							χ^2
		5	63	23	8	2	1	2		
Usefulness in life	Useful in daily life	***6***	59	26	13	1	1	1	Useless in daily life	5.7 (6) ns.
		7	56	26	14	2	1	1		
		5	37	25	14	8	5	12		
Easy-difficult	Easy to understand	***6***	34	33	19	7	4	4	Difficult to understand	16.2 (8) $p < 0.05$
		7	33	25	20	9	3	11		
		5	62	16	8	4	1	8		
Enjoyment	I want to learn it because I enjoy it	***6***	45	25	14	7	5	4	I do not want to learn it but it is a compulsory subject	22.9 (6) $p < 0.001$
		7	40	23	14	9	4	10		
		5	34	12	16	11	8	19		
Understanding-memorising	I actually understand the procedures in class	***6***	34	15	10	11	15	14	I just memorise the procedures in class	12.2 (10) ns.
		7	34	14	15	13	10	15		
		5	47	13	15	8	5	12		
Effort	I revise my lessons regularly	***6***	44	20	15	4	7	10	I revise them just before the exam or test	16.7 (10) ns.
		7	36	21	12	9	7	15		
		5	59	15	10	5	3	8		23.9 (8)
Type of revision	Enough revision at school to help	***6***	45	24	16	5	3	7	Not enough revision at school to help to help me understand well	$p < 0.01$
		7	41	26	9	6	8	9		
		5	49	20	12	5	2	12		
Extent of home help	I understand at school with little extra help from home	***6***	45	26	13	3	3	9	I understand at school with extra help from home	7.4 (8) ns.
		7	41	27	12	6	5	10		
		5	31	34	30	5	1			1.9 (4)
Kind of explanation	Diagram and pictures help me to understand mathematics well	***6***	29	37	25	6	3			ns.
		7	25	38	22	8	7			
		5	44	33	18	5	1			
Capacity to understand	I can understand the main points easily	***6***	51	33	13	2	1			9.4 (6) n.s.
		7	40	40	15	3	2			

Table 8. Areas of Difficulty (Urdu medium)

Grade 5	Grade 6	Grade 7
To draw an isosceles and a scalene triangle	Equivalent Fractions	Inheritance & Partnership
Kinds of triangles and the perimeter of a triangle	Reduction of C.F to the Simplest Form	Direct Proportion and Inverse Proportion
Kinds of angles and their measurement	Use of Brackets	Income Tax, Property Tax and Custom
To draw a perpendicular on a line	Construction of Triangles	Commission and Discount
Graphs	To Measure the Area of a Square and a Rectangle	Finding the Volume of a Cube and a Cuboid
	Graphs	Bisection of a Line Segment

Table 9. Areas of Difficulty (English Medium)

Most Difficult Topics
Distributive Property
Area
Volume
Capacity
Information Handling (basic statistics)
Algebraic Sentences
Organising and Using Data (basic statistics)
Factorising numbers into Prime Factors
Classifying Quadrilaterals
Identifying Nets for a closed Shape
Different Times Around the World
Ratio and Proportion
Collecting and Organising Data
Perimeter of Compound shapes
Reflection in a given Line
Translation, Rotation about a given point
Enlargement of 2D-Shapes
Deducing the formula for the Area of a Parallelogram
Deducing the formula for the area of a Trapezium

The curriculum for Urdu medium schools is determined nationally but English medium schools develop their own curricula. The two English medium schools each had its own curriculum. Thus, the response patterns reflect the actual specific curricula in each of the schools. As with the Urdu medium schools, geometry features highly. Statistical themes also pose problems and, again, this is almost certainly a working memory problem. In statistics, there is the added complication of statistical samples and probability as well as the use of multiple ways of presentation.

Discussion

The factor analysis reveals a number of key factors worthy of further exploration. The whole issue of memorisation and understanding is vitally important in the development of positive attitudes. The natural process for human beings is to seek to understand (Piaget**,** 1963). Jung and Reid (2009) have shown that where understanding is difficult due to working memory overloading, then the learner has to turn to memorisation and attitudes tend to deteriorate. This is a clear issue for mathematics. The goal of understanding must be stressed more if positive attitudes are to be retained.

Secondly, relating the mathematics studied to the lifestyle of the learner seems very important although ways to do this are not easy. There are two possible, and inter-related, ways forward:

(a) Using the ideas summarised in the mathematics tetrahedron (figure 1), ensure that the focus at the start is entirely on the procedures and representations. Once these have effectively been automated, the working memory is set free to consider conceptual understandings and then applications.

(b) Reduce the content covered, releasing time to develop such understandings and then to see how the mathematics learned can be applied

The correlation values obtained between performance in mathematics and working memory capacity reveal yet again the key role of limited working memory capacity in all learning, a point stressed eloquently by Kirschner *et al.* (2006). Reid (2002) showed that, at least in terms of assessment, questions can be developed which do *not* place undue stress on the working memory. The questions are still demanding but the demand lies with the understanding of the mathematics, not the capacity of the individual's working memory. It is very clear that, in Pakistan, at this age, both the curriculum and the assessment appear to be placing the working memory under stress. This is a major issue needing addressed by curriculum planners and by those who set examinations. The situation with the Urdu curriculum and assessment is particularly a matter of concern.

Key Issues

- Working Memory Capacity is known to be an issue in all subject areas but is particular important in the maths-sciences areas of the curriculum (Hindal *et al*., 2009; Reid, 2009). In the sciences, successful ways forward have been described (Hussein and Reid, 2009). It is essential that teaching and assessment in mathematics addresses this issue.
- If the cognitive load exceeds the capacity of working memory, understanding becomes a casualty and the student has to resort to memorisation, with potential attitude deterioration (Jung and Reid, 2009). This may offer a *key insight* into why there are negative attitudes towards study in mathematics.

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Appendix.

What do you think about Mathematics?

(1) What is your opinion about the **subject mathematics?**

*Tick **one** box on each line*

I like mathematics	□	□	□	□	□	□	I do not like mathematics
Useful in daily life	□	□	□	□	□	□	Useless in daily life
Easy to understand	□	□	□	□	□	□	Difficult to understand
Boring subject	□	□	□	□	□	□	Interesting subject
I do not want to learn it but it is a compulsory subject	□	□	□	□	□	□	I want to learn it because I enjoy it.

(2) What is your opinion about **mathematics lessons?**

*Tick **one** box on each line*

Easy lessons	□	□	□	□	□	□	Difficult lessons
I understand my lessons completely	□	□	□	□	□	□	I do not understand my lessons completely
I like the way my teacher explains the methods	□	□	□	□	□	□	I do not like the way my teacher explains the methods.
I just memorise the procedures in class	□	□	□	□	□	□	I actually understand the procedures in class
I do not like doing too much class work daily	□	□	□	□	□	□	I enjoy doing my class work daily
I revise my lessons regularly	□	□	□	□	□	□	I revise them just before the exam or a test.

(3) How do you feel **yourself** in your **mathematics course** at school?

*Tick **one** box on each line*

I feel I am trying hard to do well in mathematics	□	□	□	□	□	□	It is my fault I cannot study mathematics well.
I hate homework because I can't do it on my own	□	□	□	□	□	□	I enjoy homework because I can do it on my own
I am getting better at the subject	□	□	□	□	□	□	I am getting worse at the subject.
Enough revision at school to help me understand well	□	□	□	□	□	□	Not enough revision at school ... me understand well
I understand at school with little extra help at home	□	□	□	□	□	□	I understand at school only with extra help from home

(4) **Imagine** you have **problem** in understanding a **new** topic or concept. What is your likely reaction?

Tick as many boxes as you wish.

□ Start to panic	□ See it as a challenge	□ Seek help from my teacher
□ Seek help from my tutor	□ No worries, I will understand it with time.	□ Seek help from a family member

(5) Here are some descriptions of the way students **approach** mathematics.

*Tick **one** box on each line.*

	strongly agree	*agree*	*not sure*	*disagree*	*strongly disagree*
Revision sheets help me to understand mathematics well	□	□	□	□	□
Diagram and pictures help me to understand mathematics well	□	□	□	□	□
I can understand the main points easily.	□	□	□	□	□
I think mathematics help me in daily life a lot.	□	□	□	□	□
I do not want to learn mathematics but it is compulsory	□	□	□	□	□
I enjoy studying mathematics classes	□	□	□	□	□
I work hard in mathematics but cannot get good marks in exam	□	□	□	□	□

(6) Would you like to learn **more** mathematics in **next classes?**
Tick either 'yes' or 'no' and give a reason.

□ Yes, because
....

□ No, because
....

(7) I like mathematics thanks to :
*Tick **as many** boxes as you wish*

□ My parents	□ My teacher	□ My tutor	□ Mathematics lessons
□ Mathematics TV programs	□ Computers	□ My friends	□ Easy/I am good at it
□ Other - please show:			

(8) Think about **examination/tests in mathematics** at your school.
*Tick **as many** you feel true for you*

□ I tend to panic near the exam.
□ I think there is enough revision at school before exams.
□ I cannot do well in the paper because I study late night and feel sleepy.
□ I like challenging questions in exam.
□ I do not like lengthy questions because I can make more mistakes in them.
□ I find it difficult to revise the whole year syllabus in final exam.
□ The topics included in half yearly exams should not be included in final exams.
□ I do not like short questions because it does not give me chance to express that how much I know
□ I like fill in the blanks and true/false type questions in exam.
□ I like questions with colourful pictures and diagrams.
□ Sometimes I leave my questions incomplete because there is not enough space for solutions.
□ I think that the allowed time limit is very short in mathematics paper.

(9) Think of the following topics in your **mathematics syllabus**.

Easy	I understood it first time
Moderate	I found it difficult but I understand it now
Difficult	I still do not understand it

Tick the **suitable** box for each topic to show if you find that topic, easy, moderate or difficult

	Easy	*Moderate*	*Difficult*
Writing numbers in words and figures	□	□	□
Rounding off Integers	□	□	□
Number Sequences	□	□	□
Addition and Subtraction	□	□	□
Multiplication	□	□	□
Division	□	□	□

Common Fractions	☐	☐	☐
Decimal Fractions	☐	☐	☐
Percentages	☐	☐	☐
Use of Brackets	☐	☐	☐
Even and Odd Numbers	☐	☐	☐
Problem Solving	☐	☐	☐
Organising and Using Data	☐	☐	☐
Shapes and Measures	☐	☐	☐
Rectangles	☐	☐	☐
Triangles	☐	☐	☐
Symmetry	☐	☐	☐
Angles	☐	☐	☐
Recognising Parallel / Perpendicular lines	☐	☐	☐
Length, Mass, Capacity measurements	☐	☐	☐
Area	☐	☐	☐
Perimeters of rectangles and regular polygons	☐	☐	☐
Time	☐	☐	☐