Abstract: The aim of this study is to analyze and explain the mathematical connection process for students with a high mathematical ability to solve problems in terms of gender. Explorative descriptive research with a qualitative approach was used in this study. Data was collected through written tests and interviews conducted to a male and female student of class X Mathematics and Natural Sciences with high mathematical abilities. Data credibility is obtained through triangulation of methods and time. Furthermore, the data are analyzed with a flowchart which includes data reduction, data presentation, and conclusion drawing. The results showed that there were similarities and differences in the mathematical connection processes of male and female students. Similarities in the process of mathematical connections occur when making mathematical connections with other sciences and with everyday life in each of Polya's stages. In addition, the similarity of the connection process also occurs when connecting in mathematics during the re-checking stage. While the difference in the connection process in mathematics between male and female students is done at the stage of understanding the problem, solving strategies and implementing problem solving.

Keywords: Gender, mathematical ability, mathematical connections, problem solving.

Introduction

Mathematical connection is one of the most important aspects in mathematics learning (Ayunani et al., 2020; Bingolbali & Coskun, 2016; Depdikbud, 2014; Hidayati et al., 2020). This is due to mathematical connections can help students succeed with new concepts and perceived it as logical thing for daily activities (Ariyani et al., 2020; Ayunani et al., 2020). Exposure to this kind of connection adds interest and value to the study of mathematics and also broadens students’ preparation for future academic work. The relationship among elements of mathematics in the form of data, definitions, principles, operations and procedures, as well as mathematical themes and processes, shows that mathematics is not a set of isolated ideas. Students’ comprehension of the concepts becomes deeper and more lasting, when they are able to relate these mathematical concepts (Ayunani et al., 2020; Islami et al., 2018). Students learn better as they make associations with ideas and move them to long-term memory (Tout & Spithill, 2015). Therefore, students must be guided and encouraged in mathematics learning to develop the habit of thinking looking and asking about connections.

Mathematical connections are described as part of a hierarchical network, such as a web spider; an intersection or node may be viewed as part of the information contained, and a connection between nodes is a connection or relationship (Hiebert & Carpenter, 1992). Mathematical connections can also be described as schema components or schema groups that are connected in mental networks (Eli et al., 2013). The strength and compactness of a scheme depends on the connectivity of components between schema groups (Eli et al., 2013; Marshall, 1995). Mathematical connections can be grouped into two; 1) internal connections, namely connections between topics and mathematical elements, and 2) external connections, namely connections between mathematics and other subjects as well as between mathematics and everyday life (Ayunani et al., 2020; Islami et al., 2018; Kenedi et al., 2019). It means students are required to make connections between mathematical ideas, facts, procedures to understand mathematics (Hiebert & Carpenter, 1992).
Connections also help students remember skills, concepts and utilize them appropriately in problem solving (Eli et al., 2013; Hidayati et al., 2020). The problem solving process goes through several stages, namely problem understanding, planning, carrying out a plan and re-checking (Baiduri, 2013, 2018; Polya, 1973). Build connections between stages of problem solving (Tasni & Susanti, 2017) and find connections of concepts or theorems that fit the problem (Siregar & Surya, 2017) very necessary to obtain the correct settlement. The ability to solve mathematical problems has a positive effect on mathematical connection skills (Soemarmo & Hendriana, 2014). Students who have good connections can solve math problems better and vice versa.

Providing contextual treatment to students when learning mathematics will stimulate the process of mathematical connections way better than students who do not get contextual treatment (Gainsburg, 2008; Karakoc & Alacaci, 2015; Saminanto & Kartono, 2015; Swastika & Narendra, 2019) as well as with outdoor learning (Altay et al., 2017; Haji et al., 2017; Yigit Koyunkaya et al., 2018; Zengin, 2019). One of the mathematical problems that are contextual and related to daily life is compiled by the Program for International Student Assessment (PISA) which is an international student assessment program organized by the Organization for Economic Cooperation and Development (OECD).

Some research results state that the mathematical connections ability of elementary school student is still low (Hermawan & Prabawanto, 2015; Kenedi et al., 2019; Putri et al., 2016). Students’ mathematical connections in problem solving are influenced by the intelligence of mathematical logic (Islami et al., 2018). Primary and secondary student connection skills in problem solving are also influenced by mathematical abilities (Aini et al., 2016; Latif & Akib, 2016; Warih et al., 2016) as well as gender (Baiduri, 2013; Kusumaningsih et al., 2018; Zhu, 2007). Research on the ability of connections in solving gender-based mathematical problems at universities (Karim & Sumartono, 2015; Yuniawatika, 2018) which shows a different result. Therefore, efforts are needed to improve students' connection skills in learning mathematics and investigate more about mathematical connections in which male and female students have similarities and differences. Thus, the purpose of this study is to explore and describe the connection process of students with high mathematical abilities in solving PISA model problems based on gender.

**Literature Review**

*Mathematical connections in mathematical problem solving*

The mathematical connections are illustrated like a spider's web. Each node is considered pieces of information or concepts. While the series between nodes is considered as a relationship or connection. Mathematical connections are part of a information network that is interlinked with other science, which consists of essential concepts for understanding and establishing relationships between mathematical ideas, concepts and procedures (Kenedi et al., 2019). Mathematical connections are grouped into two, namely internal connections; relation between mathematical concepts and external connections; mathematical link with other sciences or in daily life (Islami et al., 2018; Orhan, 2008; Zengin, 2019).

The ability of mathematical connections expressed in the goals of school mathematics learning are: to grasp mathematical concepts, to clarify the relationship between concepts and to apply concepts or logarithms flexibly, effectively and precisely to solve problems (Leton et al., 2019). It states that the skill of mathematical connections is very important to students in relating mathematical knowledge with other sciences and in everyday life (Kenedi et al., 2019), learn mathematical concepts and problem solving (Ayunani et al., 2020; Leton et al., 2019; Mumu & Tanujaya, 2019). Students who do not understand the relationship between concepts in mathematics, will have difficulty in solving problems (Eli et al., 2013; Mumu & Tanujaya, 2019). Students still have difficulty in solving math problems (Hidayati et al., 2020).

Students are required to have problem solving skills in learning mathematics, because problem solving is the heart of learning mathematics (Barham, 2020). Solving mathematical problems will encourage students to think, foster toughness and curiosity as well as confidence to face new situations. The activity of finding answers by studying the relationships between mathematical subjects, connecting concepts in mathematics is an activity that enables students to develop their mathematical connection skills (Ayunani et al., 2020). Stages of solving mathematical problems those are very familiar, namely understanding the problem, making plans, carrying out plans, and checking again (Baiduri, 2013, 2018; Polya, 1973). This study will examine in depth the process of mathematical connections, both internal connections and external connections at each stage of problem solving.

*Mathematical ability and gender in problem solving*

Gender differences in mathematical abilities have become one of the most controversial topics in the scientific word of mathematics. Recent studies have shown that gender disparities continues to decline over the years, but the gap of ability becomes much clearer at higher levels of mathematical (Kim & Kwak, 2018). Female are less confident and tend to express strong feelings of anxiety about mathematics than male (Innabi & Dodeen, 2018). Male students are more successful than female students in math skills (Ileriturk & Kincal, 2016; Innabi & Dodeen, 2018).
Mathematical problem solving is the process of choosing the right strategy in an effort to obtain the desired answer. Capability to solve problems is an inseparable part of learning mathematics at all levels of the school (Albay, 2019; Malibiran et al., 2019). Many factors affect student performance in problem solving, including understanding processes that are built to solve problems (Malibiran et al., 2019; Voyer, 2011) and the teaching load of the teacher and gender (Malibiran et al., 2019; Osadebe & Oghomena, 2018). Gender differences in solving mathematical problems in high school and in college show that men’s performance is better than women’s (Albay, 2019; Zhu, 2007). The difference in problem solving performance is due to differences in psychological characteristics and cognitive abilities so that the strategies used are different (Baiduri, 2013; Kim & Kwak, 2018; Zhu, 2007).

Based on this fact, it can be said that the mathematical skill and gender of students will influence the method to solve mathematical problems. At the other side, efforts to find solutions to mathematical problems require skills to connect mathematical concepts both internally and externally. Therefore a hypothetical conclusion can be drawn that there are differences in the connection processes of male and female students in solving mathematical problems. This conceptual framework paper deals with the process of mathematical connection of students internally and externally in understanding problems, making plans / strategies to solve problems, implementing plans, and checking again for male and female students. This conceptual framework paper is illustrated in Figure 1.

**Figure 1. The Conceptual framework**

### Methodology

**Research Design**

The objectives of research to explore and provide an overview of the mathematical connection process of student with high ability in solving mathematical problems based on gender PISA. The type of research used is descriptive exploratory with a qualitative approach (Miles & Huberman, 1994).

**Subject and Data Collection**

The subjects of this study were two students of class X MIPA who have high mathematical abilities with different gender selected according to the results of the mathematics ability test. Students with high mathematical ability if the test results are more than 80 with a maximum score of 100 (Baiduri, 2013). The scores of the two subjects are the same, which is 85 > 80.

The main instrument in this study was the researcher, while the supporting instruments used were the recorder, the written test sheet and the interview guide sheet. The written test sheet consists of: a) adapted high school national mathematics exam questions (from multiple choice to description) of five questions with a 75 minute duration of completion as shown in Table 1. National math exam questions are used to find out mathematical abilities, to determine research subjects.

<table>
<thead>
<tr>
<th>No</th>
<th>Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Find $x$ value which satisfies the inequality $</td>
</tr>
<tr>
<td>2.</td>
<td>Determine the interval of $x$ that satisfies the inequality $\frac{4x - x^2}{x^2 + 2} \geq 0$</td>
</tr>
<tr>
<td>3.</td>
<td>Suppose $x_1$ and $x_2$ are the roots of the quadratic equation $2x^2 - 6x + 7 = 0$. Find a quadratic equation with roots $(2x_1 + 1)$ and $(2x_2 + 1)$.</td>
</tr>
<tr>
<td>4.</td>
<td>The set of solutions for the system of equations $\frac{a}{x} - \frac{14}{y} = 10$ and $\frac{6}{x} + \frac{2}{y} = 1$ is $(x, y)$. Find the value of $x - y$.</td>
</tr>
<tr>
<td>5.</td>
<td>Given the function $f(x) = \frac{2x + 3}{x - 5}$, $x \neq 5$ and $(x) = 3x + 1$. Determine the composition of the function $(gof)^{-1}(x)$</td>
</tr>
</tbody>
</table>
b) PISA model questions (mathematical connection test) as many as two questions relating to the concept of a three-variable linear equation system with connection competence. This problem is used to explore students’ mathematical connections in solving problems. One of the types of PISA question is presented below.

Eid packages for promotion and welcoming Eid, a clothing store offers several clothing packages. The package price is listed on the beside poster. The price shown on the right correspond with the row, for example in the first row, the price for a package of two t-shirts and one pair of jeans is IDR 165,000. Meanwhile, the price at the bottom of the poster shows the package price in the corresponding column. Each item can be purchased separately, but will be subject to 15% tax. If a buyer wants to buy three t-shirts and two jeans, help him to get the lowest price and show your calculations! What price must the buyer pay?

The procedure for collecting data on the connection process includes a mathematical connection test and continued interviews with the two research subjects. Settlement time is not limited with the intention that the subject explores various connections. This activity is carried out twice with different questions and times. During the process of completion and interviews recorded using a video recorder. Triangulation of methods and time is done in an effort to get credible data (Wiersma & Jurs, 2009).

Analyzing of Data

Valid data are analyzed using interactive model that includes activities (1) data reduction, (2) data presentation, and (3) conclusion drawing (Miles & Huberman, 1994). As a basis for analysis used indicators of the mathematical connection process in this research are presented in Table 2.

<table>
<thead>
<tr>
<th>Mathematical connection</th>
<th>Polya’s Stages</th>
<th>Mathematical Connection Process Indicator</th>
</tr>
</thead>
</table>
| 1. Connection in mathematics      | Understand the Problem | • Writing mathematical facts that are known to the given problem  
• Writing down what was asked  
• Identifying mathematical concepts from information on everyday life problems |
|                                   | Planning a Problem Resolution | • Finding the relationship that is asked with facts, concepts, and mathematical principles on the problem  
• Expressing mathematical procedures or operations that will be used to solve problems |
| 2. Mathematical connection with other sciences | Carry out Problem Solving | • Using connection to the planned principles  
• Applying mathematical concepts in problem solving  
• Writing with a regular pattern in solving problems  
• Using procedures according to the planned strategy |
| 3. Mathematical connections in everyday life | Re-check | • Checking facts, principles, and procedures used in solving problems  
• Checking whether the steps used are correct  
• Checking the calculation obtained |

This analysis has been carried out based on triangulation of source and time. The reliable data is obtained from two subjects which accomplished a couple of tests with a common type of answers, then proceed to the triangulation analysis of time.
Results

The purpose of this study is to explore and describe the connection process of students with high mathematical abilities in solving PISA model problems based on gender. The results of the mathematics ability test were given as many as five questions which were adapted from the high school mathematics national exam questions, so two students were selected with a score of 85 each who were categorized as students with high mathematical abilities. The results of the connection ability test using the PISA model questions are two questions and the results of interviews with the two respondents who were analyzed based on the mathematical connection process indicators in solving the PISA model problems will be presented as follows.

The Mathematical Connection Process of Female Students (P1)

Understand the Problem

The mathematical connection process undertaken by P1 students when understanding problems on the connection test answer sheet is by writing down the information obtained from the questions. The information mentioned P1 is seven Eid promo packages. The written answer to process P1 in understanding the problem is presented in Figure 2.

![Figure 2. P1 process in understanding the problem](image)

Based on the answer sheet, P1 can write problems of daily life in the form of mathematical models. P1 connects the seven packages with the mathematical concept of linear equations. It shows that P1 has made mathematical connections with daily life problems. P1 also makes connections between concepts in mathematics. It can be seen from the answer P1 when linking facts, concepts, and mathematical operations to other mathematical concepts. P1 connects variables with mathematical linear equations using addition operations.

Figure 2 shows that P1 did not write down what was asked by the questions, but from the interview data the P1 students mentioned exactly what the questions were asked. P1 also mentions if a separate purchase will be taxed. This shows that P1 made a mathematical connection with other sciences, namely economics. P1 connects the concept of price with the concept of tax. Subject P1 can also express his understanding of the problems that exist in the problem by connecting all the information that has been mentioned. This shows that P1 has made a mathematical connection in everyday life, namely by connecting the lowest price requested with the price of each package. P1 also connects with other knowledge by connecting the concept of purchasing goods separately that are taxed.

Planning a Problem Resolution

P1 devises a plan to get the lowest price requested by the problem. The plan is to find the price of each item first. Based on the interview, P1 explains that the price of each item will be calculated starting from pre- taxed to the after tax price of item. It shows that P1 makes connections between mathematics concepts, with how to connect the concepts of two variable equation system (SPLDV) and three variable equation system (SPLTV) with a mathematical procedure such as the method of elimination-substitution. P1 also associates the concept of the price of each item with the tax imposed. This P1 has made a mathematical connection with economics, when calculating the separate prices for each item. P1 also makes mathematical connections in daily life when expressing plans to find the lowest price for the item requested. When planning the lowest price purchase can also be said to do mathematical connection with other sciences. This is because the concept of price is more widely studied in economics.

Carry out Problem Resolution

The next step is to carry out the settlement. P1 does the problem according to the strategy chosen, which is looking for the price of each item. The concept of the SPLDV with the elimination-substitution method (mixed method) is used by P1 in finding the prices of each item before being subject to tax using. In addition, P1 tried many possibilities in order to find the lowest price in accordance with the question request. P1 tried approximately ten calculations using the relation of facts (price package) and the mathematical operation of addition. This activity is to determine the lowest price by comparing the results obtained. The written answer P1 is presented in Figure 3.
Based on Figure 3 and the interview, P1 has made a connection between mathematical concepts, namely by linking the system of linear equations to the method of elimination-substitution (operations and procedures) and sequences of numbers. The mathematical connection with economics is done when determining the cheapest prices, but the goods are many and look for the price of the item subject to tax, which is the base price plus tax. When P1 compares prices in an effort to get the lowest price, it means that P1 has connected the problems of everyday life with mathematical concepts.

Re-checking

The P1 process re-checks by checking the results of the calculated operations and the strategies. It means that P1 makes connections between mathematical concepts in re-checking the results obtained, namely by connecting the concept of addition and the concept of comparison (sequential numbers). This is obtained from the quotation when interviewing P1 as follows.

R: Are you sure that buying according to your calculation is the cheapest price?

P1: Because, I think the previous calculation is the cheapest, I have checked it repeatedly.

Overall, the mathematical connection process P1 in solving problems by making connections in mathematics, mathematical connections with other sciences, and mathematical connections in everyday life. At the stage of understanding the problem of the connection process in mathematics by compiling a mathematical model in the form of a system of equations. At the stage of planning a solution, the connection process in mathematics is done by planning the solution (choosing the method of elimination-substitution). Performing a solution using the elimination-substitution method and finding various alternative values to determine the minimum value is a connection process in mathematics at the problem solving stage. At the stage of re-checking the connection process in mathematics by examining the calculations and strategies used in solving problems. Connection process mathematics with other sciences (economics) when P1 links the meaning of the lowest price and tax at the stage of understanding a problem, planning a solution, and carrying out a solution.

The Mathematical Connection Process of Male Students (L1)

Understand the Problem

L1 process in understanding the problem does not appear on the written answer sheet. L1 does not write down any information about the problem, starting from presenting known and asked facts. This is due to information presented in the problem is very clear. L1 understands the problem by mentioning the important elements of the problem, namely the things that are known and asked. From the interview, L1 connecting related mathematical concepts without having to write them down. The connection component made by L1 is a mathematical connection in mathematics, a mathematical connection with problems of everyday life as well as a mathematical connection with other sciences, that is when L1 mentions the problem of finding the lowest price of the item requested for the problem. In addition, mathematical connections with other sciences by connecting separate prices with taxes.

Planning a Problem Resolution

Step L1 determines the strategy used in resolving the problem by summing up some package prices, namely package three and package five. This means that L1 has considered choosing the strategy it considers right by ignoring various other possibilities. The following is the L1 expression during the interview.
P : What strategy did you use to get the lowest price?
L1 : I chose addition of several package prices.
P : Which addition?
L1 : Addition of two package prices, package 3 and package 5.

The connection process carried out by L1 in the step of planning the completion are: 1) connections between mathematical concepts, namely by adding two packages and the lowest price, 2) mathematical connections with other sciences, namely finding the lowest price by avoiding taxes, and 3) Math connection with everyday life, namely finding the lowest price.

**Carry out Problem Resolution**

The implementation of problem solving L1 connects to the previous step, when making a solution plan. L1 works on the problem according to the specified strategy, namely by adding up the two selected packages. The completion of L1 is presented in Figure 4.

![Figure 4. L1 process in solving problems](image)

Then based on this settlement an interview with L1 was conducted. Excerpts from interviews with L1 are presented below.

R: Tell me how you got the lowest price from the problem?
L1: From the analysis that I did, package 3 consisted of 2 sugars and 1 oil, while the package 5 consists of 2 oils, 1 sugar, and 1 rice. If added together this will be 3 oils, 3 sugars, and 1 rice as desired matter.

Based on Figure 4 and interview, the mathematical connection process L1 in solving problems is: 1) connecting between mathematical concepts, i.e. selecting the addition operation followed by substituting the price of each package, 2) mathematical connection with other sciences, namely in finding the lowest price, taxes and profits/losses, and 3) mathematical connections with everyday life, which is looking for the lowest price.

**Re-Checking**

Based on interview quotes, the L1 process checks the results already obtained by checking the calculations and making sure the results obtained are the lowest prices. The lowest price because the price is not taxed. The mathematical connection process carried out by L1 in re-checking is 1) connections in mathematics, by checking arithmetic operations, 2) mathematical connections with other sciences (economics), namely determining the lowest price and tax.

The process of mathematical connection L1 in solving problems is done by connecting between concepts in mathematics, mathematical connections with other sciences in everyday life. Connections in mathematics are made at each Polya’s stage. This connection is related to understanding the lowest price, the selection of a calculation operation (addition), the price substitution of each package. Mathematical connections with other sciences are related to the lowest price, tax, and profit/loss. Whereas mathematical connections in everyday life are linked to the lowest prices.

The process of mathematical connections carried out by subjects P1 and L1 in solving problems by making connections in mathematics, mathematical connections with other sciences, and mathematical connections in everyday life. Connections in mathematics are made at each stage of Polya, namely understanding the problem, planning problem solving, carrying out problem solving, and checking the results that have been obtained.

**Discussion**

In general, the mathematical connection process carried out by male and female subjects in solving problems in this study are internal and external connections. These results are line with (Aini et al., 2016; Siagian, 2016; Siregar &
In the stage of understanding the problem, making plans and carrying out the plan of completion, female and male subjects make mathematical connections internally and externally. These results are in line with (Ajai & Imoko, 2015; Karim & Sumartono, 2015) which states there is no difference in mathematical connections and in doing mathematics between men and women. However, the connection process is different (Yuniawatika, 2018), where female subjects mathematize the given problem, by changing the problem into a system of linear equations when understanding the problem. It means that female communicates a mathematical idea into symbols, graphs, tables, or other media to explain the relationship between mathematics and problems (Orhan, 2008). Meanwhile, it is not similar to male subjects. This means that the ways of thinking and strategies undertaken by male and female subjects differ in approaching the problem (Baiduri, 2013; Kusumaningsih et al., 2018; Zhu, 2007).

In the planning stage of completion, the mathematical connection process of female subjects connects to the procedure in completing a system of three variable equations, namely by elimination-substitution (mixed method). The male subject connects by selecting two packages of goods to be added. At the implementation stage, the completion of the female subject connects to the planning stage, which is to do the elimination-substitution strategy to find the lowest price. In addition, female subjects compared various prices, including the price per unit with tax. This illustrates that the female subjects are not sure of the results obtained are the lowest price. Whereas male subjects connect with the sum of two packages of goods. Male subjects believe that the results obtained are the lowest price. This is because male subjects understand the meaning of taxable prices which are definitely more expensive. It means that there are differences in the connection process of male and female students in planning and solving problems (Gallagher et al., 2000; Yuniawatika, 2018). Differences in the mathematical connection processes of male and female students in internal connections show that gender differences in solving mathematical problems are important factors (Leder et al., 2014; Zhu, 2007). This means that internal connections can help students to integrate mathematical concepts in solving problems (Bosse, 2003).

The problem solving process requires appropriate concepts, principles and procedures in order to obtain the desired answers and is called an internal connection, connection in mathematics. The role of the teacher in learning mathematics in the classroom (Davadas & Lay, 2020; Toheri et al., 2020) and the skills of the teacher in choosing math problem solving strategies (Barham, 2020) are very important so that students are skilled in problem solving.

There are differences as well in re-checking stage made by female and male subjects (Karim & Sumartono, 2015; Kusumaningsih et al., 2018). Female subjects make mathematical connections internally, by re-checking the results obtained, namely by connecting the concept of addition and the concept of comparison (sequential numbers). Whereas male subjects connect internally and externally. Internal mathematical connections by checking calculations and making sure the results obtained are the lowest prices. Whereas the mathematical connection with other sciences (economics), namely determining the lowest price and tax.

**Conclusion**

Mathematics is built from connections of various elements; ideas, facts, concepts, principles, and procedures or operations. Mathematical connections have a very important role not only in learning mathematics and solving mathematical problems, but also in other sciences and in everyday life. The connection process in problem solving occurs between the stages of Polya completion by both female and male students with high abilities.

The internal mathematical connection process is done by both male and female students when checking back by linking it to checking the results of calculations and checking strategies. The process of internal mathematical connections of female students at the stage of understanding problems, solving strategies, and carrying out solutions is procedural, by making mathematical models, completing models, and comparing the results obtained. Whereas male students carry out internal mathematical connection processes at the stage of understanding the problem, solving strategies, and carrying out contextual solutions, namely through textual and contextual understanding of the problem, choosing the right arithmetic operations to solve the problem.

**Suggestions**

The results of this study are expected to provide new information on the mathematical relation process for students with high mathematical abilities, so that it can be used as a guide for mathematics teachers in classroom learning. Theoretically, these findings can be used as a guideline and lead to the evaluation of student mathematical interaction processes in the resolution of mathematical problems.

**Limitations and future research**

This research is limited to subjects with high mathematical ability with different genders and mathematical connections in solving PISA problems. Thus, it is still very open research for mathematical connections with other
forms of problems or in learning mathematics with various characteristics of the subject, such as low and moderate mathematics abilities, learning styles, cognitive styles, and spatial abilities.

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