

# **European Journal of Educational Research**

Volume 13, Issue 3, 1375 - 1389.

ISSN: 2165-8714 https://www.eu-jer.com/

# Student Teachers' Knowledge of School-level Geometry: Implications for Teaching and Learning

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#### Received: October 14, 2023 • Revised: December 8, 2023 • Accepted: January 4, 2024

**Abstract:** This study aimed to assess the geometric knowledge of student teachers from a university in the Eastern Cape province of South Africa. The study used a sample of 225 first-year student teachers who completed school mathematics baseline assessments on a computer-aided mathematics instruction (CAMI) software. The study adopted a descriptive cross-sectional research design, using quantitative data to measure student teachers' geometry achievement level, and qualitative data to explain the challenges encountered. The results show that student teachers exhibited a low level of understanding of school-level geometry. The low achievement levels were linked to various factors, such as insufficient grasp of geometry concepts in their secondary school education, difficulty in remembering what was done years ago, low self-confidence, and lack of Information and Communications Technology (ICT) skills along with the limited time for the baseline tests. These results suggest that appropriate measures should be taken to ensure that student teachers acquire the necessary subject-matter knowledge to teach effectively in their future classrooms.

Keywords: Computer-aided mathematics instruction, school-level geometry, student teachers, teaching and learning.

**To cite this article:** Mukuka, A., & Alex, J. K. (2024). Student teachers' knowledge of school-level geometry: Implications for teaching and learning. *European Journal of Educational Research*, *13*(3), 1375-1389. https://doi.org/10.12973/eu-jer.13.3.1375

### Introduction

Geometry is an essential component of any mathematics curriculum at all levels of education. As Jones (2002) asserts, geometry fosters mathematical reasoning, problem-solving skills, and a connection to the real world. Moreover, geometry encompasses various topics that rely on visual intuition, a fundamental human sense. According to the Royal Society (2001), knowledge of geometry helps students to remember theorems, comprehend proofs, generate hypotheses, view reality, and provide a broad perspective. Furthermore, geometry has widespread usage and application in a variety of human endeavors, such as architecture, engineering, athletics, the arts, and design (Kuzniak, 2018). Therefore, geometry is a crucial part of programs that prepare students to become teachers of mathematics.

Yet, despite its evident importance, the persistent struggles of students with geometry have been a long-standing concern at both national and global levels. This issue, as documented by various researchers over the years (e.g., Alex & Mammen, 2014; Clements & Battista, 1992; Gunhan, 2014), underscores the consistent challenge faced in geometry education. Studies consistently reveal poor performance among middle and high school students in geometry (Chen et al., 2021; Rellensmann et al., 2020). The Trends in International Mathematics and Science Study (TIMSS) 2019 outcomes further accentuate this point, highlighting the relative weakness in geometry among Grade 8 pupils across numerous countries (Mullis et al., 2020).

The struggle with geometry education is not confined to specific contexts, as indicated by research spanning various regions (Barut & Retnawati, 2020; Mullis et al., 2020; Sulistiowati et al., 2019). In sub-Sahara Africa, including South Africa, similar challenges persist among both students and teachers (Bashiru & Nyarko, 2019; Luneta, 2014, 2015; Mukamba & Makamure, 2020; Mukuka, 2023). In Burundi, evidence shows that some teachers either skip or postpone teaching geometry due to inadequate pedagogical content knowledge (Niyukuri et al., 2020). The recurrence of these issues across diverse settings emphasizes the immediate need for interventions in geometry education. This call has been highlighted by several researchers who advocate for targeted strategies to address the challenges associated with the teaching and learning of geometry (Herbst & Kosko, 2014; Smith, 2018; Ubah & Bansilal, 2019).

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A critical aspect influencing the efficacy of geometry instruction is the depth of subject-matter knowledge possessed by both current and prospective teachers (Chen et al., 2021). This expertise significantly dictates students' outcomes in geometry. Yet, Hourigan and Leavy (2017) highlight the limited amount of research investigating the geometric understanding of entry-level student teachers. This lack of exploration poses an obstacle for mathematics teacher educators in the creation of learning opportunities that would enhance preservice teachers' grasp of geometry. Therefore, this study focuses on assessing the geometry knowledge of first-year student teachers. The rationale behind this lies in the understanding that the proficiency of prospective teachers in the concepts they will be expected to teach is paramount. By evaluating the geometric knowledge of these student teachers early in their training, the study aims at equipping teacher educators with valuable insights. This, in turn, enables teacher educators to tailor interventions that enhance the expertise of prospective teachers, ultimately elevating the learning outcomes of subsequent generations of students.

This study addresses a notable gap in the existing literature by focusing on the link between the geometry proficiency of prospective teachers and the learning outcomes of students. By emphasizing the need for early interventions and improvements in teacher training programs, this research aims to pave the way for enhanced geometry education and improved student performance.

# Theoretical Perspectives on Teacher Knowledge

The significance of teachers' subject matter knowledge in effective mathematics instruction has been extensively studied (Rowland & Ruthven, 2011). Building upon Shulman's (1986) seminal work, a review by Ball et al. (2001) showed that conventional measures such as university degrees or depth of mathematics courses did not significantly correlate with student learning outcomes. Consequently, a shift was proposed towards assessing teacher knowledge based on the specific mathematical content essential for teaching, giving rise to the notion of mathematical knowledge for teaching (MKT).

According to Ball et al. (2008), MKT comprises two primary components: subject matter knowledge and pedagogical content knowledge. The former encompasses three knowledge domains—common content knowledge, specialized content knowledge, and horizon content knowledge. On the other hand, pedagogical content knowledge involves understanding the content in relation to students and teaching, as well as the curriculum. Essentially, MKT encompasses six domains of teacher knowledge.

The first one is common content knowledge, which describes mathematical skills and expertise that can be used in contexts other than teaching. Put differently, this knowledge is not confined or exclusive to the work of mathematics teachers; it can be applied in various contexts beyond the classroom.

Second, the type of mathematical knowledge that is specific to teaching is known as specialized content knowledge. Ball et al. (2008) claim that a teacher needs this kind of knowledge to respond to students' "why" questions, make connections between mathematical concepts, and come up with appropriate examples that will help students understand the material rather than just memorizing the learned concepts. Figure 1's illustration of a Grade 8 geometry question from TIMSS 2019 that asks students to find the value of an angle *x* can be used to demonstrate the distinction between common and specialized content knowledge.

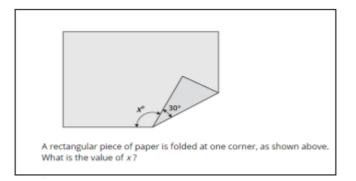


Figure 1. Grade 8 Geometry Question From TIMSS 2019 (Mullis et al., 2020, p. 197)

In this scenario, it might be tempting to assume that unfolding the folded portion of the plain paper would result in a straight angle, leading to the misconception that,  $x + 30^\circ = 180^\circ$ , which eventually leads to an incorrect value of *x* as being 150°. Another misconception that can arise here is the property that the exterior angle of a triangle is equal to the sum of two opposite interior angles, which would also lead to an incorrect calculation that  $x = 60^\circ + 90^\circ = 150^\circ$ . Hence, it requires someone to possess specialized subject-matter knowledge to anticipate these misconceptions and provide authentic learning opportunities that enable students to grasp and internalize the concept. Students can independently

discover these principles when given a blank sheet of paper to manipulate. The teacher may introduce the concepts of similarity or congruency of plane figures to facilitate a deeper understanding of the problem. Specifically, when unfolding the paper, it becomes evident that two 30-degree angles are formed, totaling 60 degrees. This insight leads to the equation  $x + 60^\circ = 180^\circ$ , yielding the correct solution of *x* being 120°.

Horizon content knowledge is the third component of the MKT model. It refers to the understanding of how mathematical concepts interconnect across the entire curriculum. This involves recognizing the relationships between different mathematical concepts, and how they build upon each other to form a cohesive and comprehensive body of knowledge. It is like having a bird's eye view of the landscape of mathematics education, where one can see how individual topics part of a larger, interconnected network are. This perspective enables teachers to plan lessons that not only focus on specific topics but also highlight their connections to other areas of mathematical concepts. This holistic approach enhances students along a path that showcases the interconnected nature of mathematical concepts. This holistic approach of isolated topics.

The fourth component relates to the fusion of content knowledge and understanding of students (i.e., knowledge of content and students). This encompasses the ability to anticipate individual student characteristics and possess a deep understanding of the mathematical concepts being taught. In the context of the example presented in Figure 1, a teacher should possess the capability to anticipate the specific challenges or misconceptions that students are likely to encounter while grasping a particular concept. Such discerning insight can only be demonstrated when a teacher possesses a comprehensive understanding of both their students and the subject matter. Additionally, this knowledge domain empowers teachers to select examples that are not only suitable for the content but also captivating and motivating for their students. It enables them to create a learning environment that resonates with their students' needs and fosters engagement in the learning process.

The fifth component pertains to the knowledge that bridges content mastery and effective teaching strategies (i.e., knowledge of content and teaching). This knowledge domain revolves around the synergy of subject matter expertise and pedagogical prowess. It is a well-established principle that one cannot effectively teach a subject they do not understand, and even if they possess profound subject knowledge, teaching success hinges on knowing how to convey that knowledge effectively (Chikiwa & Graven, 2023; Jansen, 2023).

Last but not least is the knowledge of content and the curriculum. Within this knowledge domain, understanding the intricacies of both the content and the curriculum is of paramount importance. This entails not only an understanding of the subject matter but also an awareness of how to structure and deliver that content in a manner that aligns with the curriculum objectives and meets the diverse needs of learners. In essence, this knowledge empowers teachers to bridge the gap between subject matter expertise and the art of teaching, ensuring that students receive a well-rounded and meaningful educational experience.

This MKT framework has become a cornerstone in mathematics teacher education research, including geometry instruction (Herbst & Kosko, 2014; Smith, 2018). It emphasizes a dual foundation, suggesting that teachers should be well-versed not only in higher-level college mathematics but also in the foundational concepts taught in schools (Nason et al., 2012). Despite this theoretical ideal, many student teachers struggle with acquiring adequate subject matter knowledge not only in geometry but other mathematical concepts, as evidenced by numerous studies across diverse settings (Adelabu & Alex, 2022b; Bowie et al., 2019; Malambo et al., 2018; Saili et al., 2023).

These studies consistently reinforce the need for prospective teachers to possess robust subject matter knowledge in what they will teach upon professional qualification (Malambo et al., 2018; Niyukuri et al., 2020). This resounding chorus of research accentuates a pivotal truth: teachers must command a deep understanding of the concepts they are entrusted to teach. Moreover, this foundational knowledge is not merely desirable but imperative in shaping effective teaching practices and fostering comprehensive student learning experiences.

In addition to highlighting the importance of subject matter knowledge, some researchers have also focused on developing assessment tools to measure teachers' mathematical knowledge in geometry. For instance, Martinovic and Manizade (2018) directed their research towards devising assessment tools tailored to gauge teachers' mathematical knowledge specifically in geometry instruction. Their proposal advocated for the creation of targeted "probes" centered on topics commonly taught within the geometry domain. Consistent with this call, this paper aims to evaluate student teachers' proficiency in specific areas of geometry, aligning closely with the demands and content typically covered in school-level geometry.

# Scope of the Study

This article is located within the research domain that attempts to understand and describe student teachers' content knowledge in school level geometry. According to Alex and Trow (2023), preservice teachers ought to possess sufficient levels of mathematical knowledge for them to teach effectively. This paper reports on such evidence gathered from the school level geometry test written by the first-year student teachers. The test items were administered through the

computer aided mathematics instruction (CAMI) software. CAMI is a licensed computer programme (available at http://www.cami.co.za/) that can be used for mathematics baseline assessments in schools and teacher training institutions. According to earlier research by Adelabu and Alex (2022a) and Alex and Trow (2023), CAMI is a highly productive software that has a great deal of potential for raising student teachers' test scores in mathematics with little guidance from their instructors.

As indicated earlier, the teaching and learning of geometry has been identified as an area that poses various challenges for students and teachers at different educational levels. This study was guided by mathematical knowledge for teaching, a model developed by Ball et al. (2008). This model has been applied in different aspects of mathematics education research including the assessment of mathematical knowledge for teaching geometry (MKT-G) by Smith (2018). The following research question was examined:

# What evidence can be drawn from the first-year student teachers' performance on tasks involving secondary school geometry?

The expectation is that by addressing this question, we could establish a foundation for interventions designed to enhance the understanding of school-level geometry among student teachers. This is of paramount importance as they will be required to impart this knowledge once they embark on their teaching careers. Furthermore, the insights gained could serve as a valuable resource for reforming the initial mathematics teacher training curriculum. This would ensure that our future teachers are well-equipped and confident in teaching geometry, ultimately leading to improved student learning outcomes. The potential impact of these interventions and curriculum reforms extends beyond individual classrooms, potentially influencing the broader educational landscape and contributing to the advancement of mathematical proficiency among students nationwide. Therefore, it is not just an anticipation but a necessity to provide answers to the above question.

#### Methodology

#### Research Design

In this study, a descriptive cross-sectional research design was utilized. This design was chosen due to its ability to facilitate the collection of both quantitative and qualitative data, thereby providing an understanding of student teachers' content knowledge in school-level geometry. The quantitative aspect of the data collection provided valuable insights into the achievement levels of the student teachers. It allowed for an objective measurement of their understanding and proficiency in geometry, offering a clear picture of their academic performance in this specific area. On the other hand, the qualitative data gathered offered a deeper exploration into the challenges faced by the student teachers. Through their feedback, we were able to identify specific geometry concepts that they found particularly difficult. This qualitative approach provided a platform for the student teachers to express their thoughts and experiences, thereby shedding light on potential reasons behind their difficulties with certain geometry concepts.

#### Study Participants and Setting

This paper reports part of the findings of ongoing research being conducted on the different aspects of mathematical knowledge for teaching among Senior and FET Phase student teachers in a Bachelor of Education (BEd) program. The study is just one of the many tasks carried out by the Mathematics Education and Research Centre, which was founded at a rural university in South Africa's Eastern Cape province. The focus of this paper is solely assessing the performance of first-year student teachers in geometry-related concepts, even though the entire project focuses on every topic taught in schools from Grade 7 to Grade 12. Geometry was specifically chosen following previous studies that have reported teachers' and students' difficulties in handling the topic both locally and internationally (Alex & Mammen, 2014, 2018; Bashiru & Nyarko, 2019; Luneta, 2014, 2015; Mukuka, 2023; Mukamba & Makamure, 2020; Niyukuri et al., 2020). In the "results" section, specific geometry concepts upon which the analysis is based are stated.

Purposive sampling technique was used to select the participants. This sampling technique was appropriate in the sense that such baseline assessments are usually administered to first year student teachers of mathematics. The intention was to create a profile of first year student teachers' understanding of school-level mathematics. This profiling aimed to equip teacher educators with essential insights, allowing them to craft targeted interventions to address any identified inadequacies in the understanding of various school-level mathematical concepts. Although all 262 students enrolled in the Senior and FET Phase Mathematics Education programme were the focus of the main research project, it suffices to point out that not all of them attempted all six baseline assessments (from Grades 7 to 12). As a result, we centered our analysis on the student teachers who took each test. The "results" section lists these varying sample sizes along with the findings. Specifically, the number of student teachers who undertook the 6 baseline tests ranged from 156 to 255.

Furthermore, after the baseline assessments, 74 student teachers voluntarily participated in a post-assessment survey. This subset of participants revealed a gender distribution with 48.6% male (n = 36) and 51.4% female (n = 38). Nonetheless, it is worth noting that this gender breakdown provides a snapshot of the demographics within the smaller

subset that participated in the post-assessment survey, not necessarily reflective of the broader population of student teachers in the program.

The three baseline tests for the General Education and Training (GET) phase (Grades 7-9) and the baseline tests for the Further Education and Training (FET) phase (Grades 10–12) each took three hours to complete for each student. However, extra time of up to 30 minutes was given for each session to allow trainee teachers to make up any delay that could have occurred. At the outset, it was promised to student teachers that their scores on these tests would not contribute to their grades in the mainstream courses. However, student teachers were informed of the importance of engaging in such an activity in that their performance would help identify their areas of difficulty with regards to school mathematics.

Baseline testing was conducted using the CAMI software. This process was carried out over a two-week period, a duration necessitated by the limited availability of computer lab access for all student teachers. Since the CAMI program draws its content questions from the Curriculum and Assessment Policy Statement (CAPS) document, the items' reliability and authenticity were guaranteed. This ensures that every test item was both reliable and authentic, thereby eliminating the need for additional reliability and validity checks. It is also crucial to highlight that the CAMI baseline testing encompasses both high-order and low-order cognitive level items. This approach ensures a balanced evaluation of students' abilities across a range of cognitive levels. Figures 2 and 3 provide sample test items from Grades 9 and 12 respectively, offering a glimpse into the depth and breadth of the testing material.

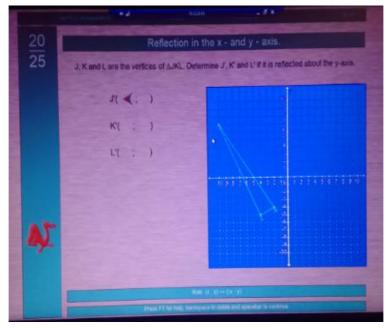


Figure 2. Sample Question From Grade 9 Baseline Test

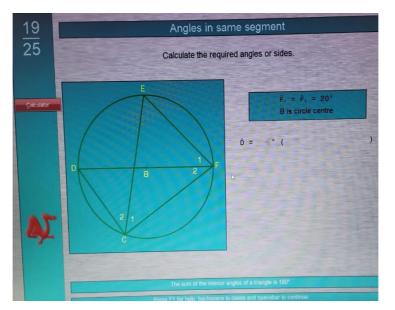


Figure 3. Sample Question From Grade 12 Baseline Test

# Data Collection

We extracted test results from the CAMI software, which automatically generates performance reports in various formats. We used individual reports for each student and grade to obtain feedback on each question. The feedback included the content coverage, the student's performance, and the time taken to complete the test. The software scored each question as 1 for correct and 0 for incorrect answer. We used the reports to determine how many student teachers attempted each test and how many answered each question correctly in all six baseline tests. We also conducted a feedback survey using Google Forms, with 74 student teachers participating. The survey asked about their experiences and reflections on using the software and the test questions. For this paper, we focused on analyzing which geometry topics they found challenging and why.

# Data Analysis

We used frequency distributions to quantify the proportion of correct responses for each question, providing an overview of the topics that student teachers found challenging versus those they found manageable. We analysed the feedback survey data qualitatively, with a focus on submissions from student teachers who identified geometry as problematic, along with their reasons. This analysis offered deeper insights into areas of concern related to student teachers' difficulties with school-level geometry. Although our approach to analyzing qualitative data did not strictly adhere to the steps for conducting thematic analysis as outlined by Braun and Clarke (2006) and Nowell et al. (2017), we selected excerpts from student teachers' survey responses that aligned with our quantitative findings. In essence, the quantitative data helped determine the student teachers' achievement level in geometry, while the qualitative data aided in interpreting the challenges associated with poor achievement. In the context of primary school teachers, a minimum mastery threshold of 60% was applied (Venkat & Spaull, 2015). This proficiency level was deemed suitable for our study, given that student teachers were being evaluated on concepts they had previously encountered.

# Ethical Clearence

Participation in the baseline tests was entirely voluntary. An explanation of the study's purpose was provided to the participants, highlighting its importance for both the research centre and the university in gauging the student teachers' proficiency in various school mathematics topics. By participating in the baseline assessments and the subsequent feedback survey, the student teachers consented to have their responses analysed and published. Prior to administering the tests, the researchers obtained ethical clearance from a relevant ethical committee at the university where the study was conducted (ref no. FEDSRECC001-06-21).

#### **Findings/Results**

Results of student teachers' performance on geometry related concepts are grouped according to the grade levels for two phases namely General Education and Training (GET) and Further Education and Training (FET). The concepts on which the analysis is centered are clustered within six geometry content areas namely, measurements, space and shape, transformation geometry, analytic geometry, circle geometry, and trigonometry. For each grade level, student teachers were expected to attempt 25 questions in total (except for Grade 7 that comprised 20 questions).

#### Grade 7 Baseline Test

At Grade 7 level, we analysed student teachers' responses to 6 questions on geometry related concepts. Out of the targeted 262 student teachers, 225 participated in the Grade 7 baseline test. The frequency and percentage of correct responses among those who took part are shown in Table 1.

| Table 1. Proportion of Correct Responses at Grade 7 Level Geometry |
|--|
|--|

| Topic coverage                     | Ν   | Frequency | Percent |
|------------------------------------|-----|-----------|---------|
| Identifying lines of symmetry      | 225 | 32        | 14.2    |
| Identifying type of an angle       | 225 | 130       | 57.8    |
| Angle of rotation on the clock     | 225 | 129       | 57.3    |
| Volume of a geometric figure       | 225 | 80        | 35.6    |
| Coordinates on the Cartesian plane | 225 | 120       | 53.3    |
| Perimeter & area                   | 225 | 1         | 0.44    |
| Average                            |     |           | 36.4    |

Results displayed in Table 1 show that student teachers' average performance on geometry-related test items was 36.4%. which is below the expected standard of 60%. The least performance was recorded on measurements with regards to the concept of perimeter and area on which only 1 (0.44%) of the 225 respondents provided a correct answer. When asked which topics in the Grade 7 baseline Test were difficult for student teachers to answer, and why they felt this way, the following reasons were given:

*ST57:* I had challenges with questions on measurements because it has been long, and they were never my favourite.

ST62: Measurements because they are hard to understand.

ST72: Geometry because I did not have a good background of it.

The reasons cited above are a clear demonstration that some difficulties encountered by student teachers could be traced from their failure to understand the concepts during their secondary school education.

# Grade 8 Baseline Test

Grade 8 baseline test was attempted by 218 student teachers. Out of 25 questions tested at this level, five were based on geometry-related concepts (mainly space and shape and transformation geometry). Table 2 illustrates the frequency and percentage of the student teachers who answered each of the five questions correctly.

| Topic coverage           | Ν   | Frequency | Percent |
|--------------------------|-----|-----------|---------|
| Properties of 2-D shapes | 218 | 84        | 38.5    |
| Properties of 3-D shapes | 218 | 89        | 40.8    |
| Classification of angles | 218 | 99        | 45.4    |
| Translations             | 218 | 32        | 14.7    |
| Enlargement              | 218 | 16        | 7.34    |
| Average                  |     |           | 29.4    |

Table 2. Proportion of Correct Responses at Grade 8 Level Geometry

Based on the results displayed in Table 2, the average proportion of those who answered geometry questions correctly was 29.4%, which is far below the expected standard of 60%. The lowest was recorded on enlargement (7.34%) whereas the classification of angles received the highest number (99 or 45.4%) of correct responses. In response to a follow-up feedback survey that requested them to indicate the most difficult questions they encountered and to give reasons why such questions were difficult, the following submissions were made:

*ST17*: Area and perimeter since I had forgotten how to calculate.

ST42: Geometry because I did not prepare myself for it.

*ST46*: Geometric transformations since I had forgotten how they are done.

ST72: Geometry because I didn't have a good foundation in previous years.

Based on the reasons advanced by student teachers, it seems clear that they believed they could not manage to answer certain questions correctly because they had forgotten certain basic concepts.

# Grade 9 Baseline Test

At Grade 9 level, 156 student teachers completed the test. There were seven questions on geometry–analytic geometry, space and shape, and transformation geometry. Table 3 displays the frequency and percentage of correct responses to each of the seven geometry related questions.

| Topic coverage                        | Ν   | Frequency | Percent |
|---------------------------------------|-----|-----------|---------|
| Cartesian coordinate plane            | 156 | 75        | 48.1    |
| Angles associated with parallel lines | 156 | 28        | 17.9    |
| Introduction to triangles             | 156 | 53        | 34.0    |
| Volume                                | 156 | 3         | 1.92    |
| Reflections                           | 156 | 90        | 57.7    |
| Translations                          | 156 | 28        | 17.9    |
| Enlargement                           | 156 | 10        | 6.41    |
| Average                               | •   |           | 26.3    |

Table 3. Proportion of Correct Responses at Grade 9 Level Geometry

Based on the results displayed in Table 3, the question concerning volume determination yielded the lowest correct response rate, with only 3 participants (1.92%) answering correctly. Conversely, the Cartesian coordinate plane question garnered the highest correct responses, with 75 participants (48.1%) getting it right. Overall, the average performance across all topics presented in Table 3 stood at 26.3%. In response to the feedback survey, *ST10* indicated that calculating volume was one of the questions he/she had been omitting because he/she never liked the topic. Here, it suffices to point out that never liking a topic, resulting in skipping questions on that topic during tests could have been influenced by many factors, one of which should relate to the way the topic was taught.

#### Grade 10 Baseline Test

This test was attempted by 213 student teachers. Of the 25 test questions, five of them were based on geometry related concepts as shown in Table 4.

| Topic coverage                          | Ν   | Frequency | Percent |
|---|-----|-----------|---------|
| Measurement - perimeter                 | 213 | 50        | 23.5    |
| Measurement - area                      | 213 | 66        | 31.0    |
| Measurement -volume                     | 213 | 3         | 1.41    |
| Measurement - volume                    | 213 | 91        | 42.7    |
| Circle geometry and congruent triangles | 213 | 74        | 34.7    |
| Average                                 |     |           | 26.7    |

Table 4. Proportion of Correct Responses at Grade 10 Level Geometry

Student teachers' performance on geometry-related concepts in the Grade 10 baseline test (Table 4) fell below expectations, with an average of only 26.7% correctly answering the questions. Notably, the lowest achievement level (1.41%) was observed in a question focusing on volume calculations. These outcomes closely resemble those displayed in Table 3 for the Grade 9 baseline test. During a feedback survey, student teachers gave different reasons regarding the difficulties they faced in geometry at this level:

**ST7:** Geometry; I can't lie; I don't understand it at all.

**ST16**: Geometry; because of the carelessness I had in that grade.

ST30: Geometry; it is hard to understand it.

**ST55**: Geometry; it was difficult for me to press angles on the computer.

The response given by student teacher (ST55), is also an indication that the struggle could be attributed to the lack of Information and Communications Technology (ICT) skills coupled with time limitations.

# Grade 11 Baseline Test

The Grade 11 baseline test was attempted by 211 student teachers. Six out of the 25 questions were based on concepts related to geometry (three on trigonometry and two on analytic geometry and one on space and shape). Table 5 illustrates the proportion of student teachers who answered each of the six geometry questions correctly.

| Topic coverage                            | Ν   | Frequency | Percent |
|---|-----|-----------|---------|
| Trigonometry (reduction & special angles) | 211 | 19        | 9.00    |
| Trigonometry (reduction & special angles) | 211 | 44        | 20.9    |
| Trigonometry (solutions of triangles)     | 211 | 33        | 15.6    |
| Space and shape (mid-point theorem)       | 211 | 21        | 9.95    |
| Analytic geometry (gradient)              | 211 | 27        | 12.8    |
| Analytic geometry (midpoint)              | 211 | 46        | 21.8    |
| Average                                   |     |           | 15.0    |

Table 5. Proportion of Correct Responses at Grade 11 Level Geometry

Performance at this level was much lower than that of grades 7 to 10 as only 15.0% of the 211 student teachers managed to answer geometry questions correctly. The lowest achievement rate (9.00%) was recorded on trigonometry (reduction and special angles) while the highest (21.8%) was recorded on analytic geometry. The following submissions from student teachers are a true reflection of this level of achievement:

**ST37:** Trigonometry because I could not press the computer well although I knew the answer.

ST40: Trigonometric functions are difficult, and they need a lot of time.

**ST54**: Trigonometry; since when I was young, it was always a tricky topic to understand.

ST68: Trigonometry; it happens that I have already forgotten some rules that are required on it.

ST72: Geometry because I did not have a good background of it.

The above quotes from student teachers are a clear demonstration that trigonometry is one of the areas where they faced serious challenges not only during this baseline test but also during their high school days.

# Garde 12 Baseline Test

This test was attempted by 205 student teachers. Results displayed in Table 6 reflect that more than half of the test questions were based on geometry related concepts. Of the 15 geometry questions, 12 were based on trigonometry, one on analytic geometry and two on space and shape (circle geometry). Results displayed in Table 6 show that student teachers' performance on Grade 12 geometry related concepts was the least of all the six grade levels examined. On average, only 12.6% of the responses to geometry items were correct. This average performance is a true reflection of the low achievement levels recorded on each of the 15 questions ranging from 0 on trigonometry (sine rule) to 36.6% on another trigonometry question.

| Topic coverage                                      | N   | Frequency | Percent |
|---|-----|-----------|---------|
| Trigonometry (equation)                             | 205 | 13        | 6.34    |
| Trigonometry (diagram question)                     | 205 | 75        | 36.6    |
| Trigonometry (equation)                             | 205 | 21        | 10.2    |
| Trigonometry (equation with unusual interval)       | 205 | 3         | 1.46    |
| Trigonometry (equation with negative values)        | 205 | 9         | 4.39    |
| Trigonometry (solutions of right-angled triangles)  | 205 | 17        | 8.29    |
| Trigonometry (solutions of triangles -sine rule)    | 205 | 0         | 0.00    |
| Trigonometry (solutions of triangles - cosine rule) | 205 | 31        | 15.1    |
| Trigonometry (solutions of triangles - area)        | 205 | 44        | 21.5    |
| Trigonometry (graphs -horizontal shift)             | 205 | 49        | 23.9    |
| Trigonometry (graphs - vertical shift)              | 205 | 21        | 10.2    |
| Trigonometry (identities)                           | 205 | 2         | 0.98    |
| Analytic geometry                                   | 205 | 43        | 21.0    |
| Space and shape (circle geometry)                   | 205 | 42        | 20.5    |
| Space and shape (circle geometry)                   | 205 | 18        | 8.78    |
| Average   |     |           | 12.6    |

Majority of the responses to the feedback survey indicated that they struggled with geometry questions (and trigonometry in particular) due to poor foundational knowledge of the topic while others cited the issue of failing to cope with the computer when it came to pressing of angles. For instance, some participants indicated that it was hard for them to deal with trigonometry on the computer, while others claimed that trigonometry was not their favourite.

#### Student Teachers' Overall Performance in Geometry

Generally, student teachers' performance on geometry related concepts was below the 60% expected masterly level, ranging from 12.6% ( $\approx$  13%) at Grade 12 level to 36.4% ( $\approx$  36%) at Grade 7 level as shown in Figure 4. Results displayed in Figure 4 further indicate that the higher the grade level, the lower the proportion of correct responses with regards to geometry related concepts.

The most cited reasons for the difficulties student teachers faced in handling questions on geometry related concepts include lack of practice on geometry, low self-confidence, lack of ICT skills coupled with the limited time within which the test was administered, inadequate understanding of geometry concepts during student teachers' secondary school days, failure to recall what was done some years back, and inadequate preparation time, among others.

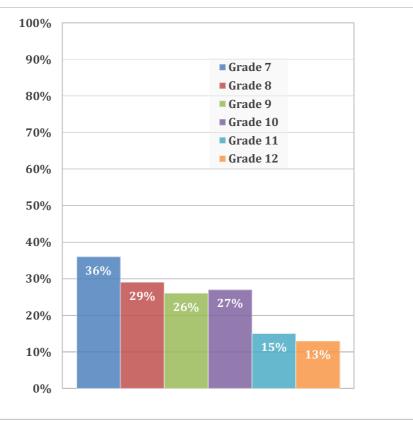


Figure 4. Student Teachers' Overall Performance in Geometry Baseline Tests

# Discussion

Findings of this study have shown that student teachers performed poorly on tasks involving secondary school geometry. The student teachers' average scores for each of the six baseline tests were significantly below the minimal mastery level of 60%, which we adopted from the PrimTEd project by Venkat and Spaull (2015). In addition, our analysis shows that student teachers' performance ranged from 1 (not achieved) to 2 (elementary achievement) on the 7-point scale used in the Curriculum and Assessment Policy Statement (CAPS) (Department of Basic Education, 2011), indicating a fundamental struggle with geometry concepts.

The consistent underperformance observed among student teachers in this study re-affirms the prevalent struggles encountered with geometry concepts, as documented in prior studies across various contexts (Alex & Mammen, 2014, 2018; Bashiru & Nyarko, 2019; Luneta, 2014, 2015; Niyukuri et al., 2020; Trimurtini et al., 2022). For instance, a study by Hourigan and Leavy (2017) brought to light a deficiency in geometric thinking among 50% of participants (student teachers), casting doubt on whether pre-university experiences adequately prepared them for smooth entry into mathematics programs for teaching. Similarly, research by Couto and Vale (2013) pointed to unsatisfactory performance by prospective mathematics teachers in elementary geometry knowledge assessments. Another investigation by Romano (2017) on preservice teachers' understanding of fundamental geometric concepts, their comprehension of the processes involving these objects fell significantly short.

Moreover, it has been noted that these challenges are not confined to preservice teachers alone; in-service teachers also grapple with similar issues. For instance, a study by Sunzuma and Maharaj (2019) in Zimbabwe, revealed that even experienced teachers bypassed certain geometry topics due to insufficient mastery to teach them effectively. It was found that approximately 47.5% of in-service teachers lacked the readiness to teach geometry due to insufficient competency in the subject matter. This finding resonates with Tachie's (2020) study in South Africa, exposing a prevalent inadequacy in both content knowledge and pedagogical approaches related to teaching geometry among the majority of participating teachers. This pattern underscores a critical issue: the difficulties students face in school geometry persist through preservice teacher training. Consequently, if left unaddressed, these deficiencies continue to manifest even after teachers attain their professional qualifications. This cycle perpetuates a scenario where students' struggles with geometry might be directly linked to their teachers' inadequate grasp of these concepts.

Findings of the present study have also shown that some student teachers struggled with computer usage, especially for tasks related to angles. This situation could largely be due to the fact that these participants were relatively new to the university system, and some might have had minimal exposure to computers during their high school education.

Although this lack of exposure might have contributed to their low performance, it doesn't fully explain it. The CAMI software offers guidance on how to perform specific tasks involving a variety of mathematical notations and symbols. Hence, these challenges underscore the deficiencies in ICT competency among student teachers. Earlier, Sinclair and Bruce (2015) emphasized that one way to improve geometry teaching and learning is using digital technology. The use of digital technology, such as GeoGebra, has been suggested as a method to improve geometry teaching and learning (Bayaga et al., 2020; Mukamba & Makamure, 2020; Uwurukundo et al., 2022). This emphasizes the need for teacher education programmes to ensure that prospective teachers are equipped with the knowledge and skills to effectively use digital technology in their future classrooms.

Furthermore, student teachers expressed low confidence in their ability to answer geometry questions. For instance, one student teacher stated: *"Trigonometry and geometry; those are the topics which beat me even if I try to pull up my socks regarding them, the socks pull me down instead."* If not addressed, this lack of confidence may lead to a negative mindset about geometry and mathematics in general (Ukobizaba et al., 2021). This aligns with a study by Mukuka et al. (2021), which highlights the importance of interventions aimed at boosting students' self-confidence and enthusiasm for learning mathematics.

Overall, the inadequate grasp and lack of confidence among student teachers when it comes to handling geometry concepts can be traced back to their limited understanding during secondary education. For instance, Romano (2017) emphasized that the struggles encountered by student teachers in geometry were not due to inherent intellectual shortcomings but were predominantly the result of inadequacies in the foundational components of geometric thinking developed during their earlier schooling years. This recognition has prompted numerous researchers, such as, Mukuka and Alex (2024), and Niyukuri et al. (2020), to highlight the pressing need of ensuring that student teachers possess a thorough command of the subject matter they are expected to teach before stepping into the classroom. This emphasis aims to address the root cause of challenges faced by teacher educators and underscores the importance of solidifying foundational knowledge to enhance teaching efficacy.

# Conclusion

The findings of this study highlight significant challenges in the current state of teacher education, particularly in the areas of geometry and ICT competency. The poor performance in geometry tasks and the fundamental struggle with geometry concepts among student teachers reveals a critical gap in their mathematical foundation. This is further compounded by their self-reported lack of understanding of certain geometry concepts during their secondary school education. Moreover, the deficiencies in ICT competency among student teachers, despite the guidance offered by the CAMI software, suggest a need for a more robust integration of ICT skills in teacher training programs. The findings of this study have implications for the design and implementation of teacher education programs that aim to equip student teachers with the necessary knowledge and skills to teach geometry effectively. The study suggests that student teachers need more opportunities to develop their geometric thinking and reasoning, as well as their ICT competency, through engaging and challenging activities that foster conceptual understanding and problem-solving. The findings of this study further have implications for the policy and practice of mathematics education in South Africa and beyond. The study reveals a gap between the expectations and realities of geometry education, as student teachers struggle to meet the standards and outcomes set by both the national curriculum (CAPS document) and the research community. The study urges for more support and guidance for student teachers and in-service teachers, as well as more collaboration and communication among stakeholders, such as universities, schools, and education authorities, to address the challenges and improve the quality of geometry education.

#### Recommendations

Considering the findings of this study, the following recommendations are made.

The first one pertains to the improvement in geometry instruction. The study suggests that student teachers exhibited a low level of understanding of school-level geometry. Therefore, it is recommended that there be a focus on improving the quality of geometry instruction in teacher education programs.

The second is a focus on addressing conceptual gaps in geometry. The low achievement levels were linked to insufficient grasp of geometry concepts in their secondary school education. This implies the need to address these conceptual gaps in the curriculum and provide additional support to students who struggle with these foundational concepts.

The third is a need for remedial education. The study indicates that student teachers have difficulties in remembering what they learned years ago. This suggests the need for remedial education or refresher courses to help student teachers review and reinforce their geometry knowledge.

The fourth recommendation pertains to confidence building. Low self-confidence was identified as a contributing factor to low achievement levels. Thus, measures should be taken to boost the self-confidence of student teachers, possibly through supportive and confidence-building activities.

Lastly, arising from the methodological limitations in the current study (as highlighted in the next section), it is recommended that future studies could consider a more detailed analysis of the specific areas where student teachers make mistakes in their solutions. This could provide insights into whether the errors are computational or conceptual, which would be valuable for designing targeted interventions. The impact of technological familiarity on student achievement could also be explored further. Future research could investigate the extent to which lack of computer skills affects performance on tasks administered via software like CAMI. This could inform the development of support measures to improve digital literacy among student teachers. Additionally, future studies could collect more comprehensive demographic data, such as socio-economic backgrounds or prior educational experiences. This would allow for a more nuanced understanding of the factors that influence performance in geometry tasks. Finally, longitudinal studies could be conducted to track the progress of student teachers over time. This could provide insights into how their geometry knowledge and teaching skills develop throughout their teacher education program and into their professional careers.

#### Limitations

This study, while valuable, had certain limitations. Primarily, the study relied on data generated from the CAMI software. This approach did not allow for a detailed analysis of the specific areas where student teachers made mistakes in their solutions. It is possible that some errors could have been computational rather than conceptual.

Another potential barrier to student achievement was a lack of familiarity with computer usage. The administration of baseline tests via the CAMI software may have disadvantaged some participants who were not technologically adept, thereby affecting their scores.

While the sampling approach offered depth in assessing knowledge levels across various mathematical concepts, the absence of comprehensive demographic data, beyond names and gender in the survey, could limit the ability to draw nuanced conclusions or to understand potential correlations between performance and other factors like socio-economic backgrounds or prior educational experiences. This limitation should be acknowledged when interpreting the findings and considering potential interventions for addressing any identified inadequacies in mathematical knowledge among these first-year student teachers.

#### Acknowledgement

We would like to sincerely thank the student teachers who took part in this study.

# **Authorship Contribution Statement**

Mukuka: Conceptualization, design, data analysis/interpretation, drafting manuscript. Alex: Conceptualization, editing/reviewing, supervision, final approval.

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