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Analysis of Mathematics Lesson Planning Framed by the Teachers' Pedagogical Content Knowledge

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Abstract: Lesson planning is considered to be an important and efficient tool for effective teaching and learning process. Preparations of effective lesson plan requires teachers to be competent in the pedagogical content knowledge (PCK). This study investigated how the mathematics teachers' PCK impact their competences on designing effective lesson plans. Twelve in-service mathematics teachers from public secondary schools in Unguja-Island (Zanzibar) of Tanzania were involved. The data were collected using the reviewing of the teachers' mathematics lesson plan documents and teachers' interview. The data were analyzed in both quantitative and qualitative mode based on the lesson plan framework guideline (LPFG) and the criteria for better planning of the sections jointly with PCK abilities indicators. The percentage average of occurrence of the sections in the teachers' designed lesson plan (DLP) and the percentage of occurrence of different category within the sections of the DLP in relation to the criteria were determined. Also, the challenges faced by the teachers in their DLP were identified. The result of the analysis revealed that the PCK competences of mathematics teachers are not good enough to impact their DLP, as some of their designing lesson plan sections were occurred inconsistently. Also, the implementation of PCK competences for effective mathematics teaching found to be at developing stage, as some criteria related to it were not clearly observed in their DLP. The study recommends the demand of in-service training for mathematics teachers on the implementation of teachers' competences particularly PCK in the lesson plan designing for effective classroom practices.

Keywords: *Designed lesson plan, effective mathematics teaching, in-service mathematics teachers, pedagogical content knowledge.*

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Introduction

Effective mathematics teaching is of hot debate in this contemporary world. Many recent studies investigated different ways that can support mathematics teachers to provide successful teaching and meaningful learning to students. According to Ika et al. (2017), teaching is a task that requires teachers to have good and effective preparation and plan of their lesson. It also demands teachers to have knowledge and skills in the delivery of a particular lesson. Therefore, sufficient understanding of content knowledge, appropriate teaching strategies, together with skills in organizing and integrating the lesson focusing on how learners are essential tools for good and quality mathematics teacher (Danisman & Tanisli, 2017). These entire components of knowledge for teachers refer to the notion of pedagogical content knowledge (PCK).

PCK is among the fundamental domain knowledge that is directly connected to the action of teaching. This type of knowledge is mostly used by mathematics researchers to explain the relationship between mathematics content, pedagogy and learners (Karim & Danaryanti, 2020). Mathematically, PCK is observed to be the teachers' knowledge of teaching mathematics topics regarding the learners' difficulties, preconception and misconception (Turnuklu & Yesildere, 2007), thereby allow teachers to represent mathematics concepts in multiple ways, use varieties and real-life examples and apply different strategies regarding the students' characteristics and differences Moh'd et al. (2021). "Pedagogical content knowledge identifies the distinctive bodies of knowledge for teaching. It represents the blending of content and pedagogy into an understanding of how particular topics, problems or issues are organized, represented,

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and adapted to the diverse interests and abilities of learners, and presented for instruction " (Shulman, 1987, p.8). Thus, PCK for mathematics teachers is regarded as an appropriate knowledge for better plan of teaching.

The review of literature reveals that insufficient understanding of PCK for teachers are amongst the factors which leads to poor designing of the lesson plan (Ika et al., 2017). PCK is the knowledge which signifies effective learning as a result of integrating different kinds of knowledge for teaching particular content (Ika, et al., 2017; Ma'rufi, 2015), hence it is the knowledge which is required by mathematics teachers in their entire preparation of the lesson plan from the selection of appropriate competences to the evaluation procedure (Aimah et al., 2019; Ika et al., 2017). Lack of sufficient PCK knowledge result in the failure of setting and/or achieving the realistic goal, the attainment of the competences in terms of knowledge and skills, choosing range and appropriate teaching strategies that complement the topic being taught, selecting and mastering teaching material and resources as well as designing different students' activities that are used for assessing their understanding.

The revolution government of Zanzibar through the Ministry of Education and Vocational Training (MoEVT) under Zanzibar Institute of Education (ZIE) has provided the guideline for teachers' records keeping system, of which the lesson plan is amongst. This guideline is normally used by all secondary school teachers (Ministry of Education [MoE], 1995) and was reviewed regarding the transformation that teachers undergo from content to competency based (MoEVT, 2016), as well as to reflect the demand of the secondary school curriculum of Tanzania (MoEVT, 2007). In this study researchers used the lesson plan framework guideline (LPFG) provided (please see Appendix A), as the base for investigating how the mathematics teachers' PCK impact their competences on designing effective lesson plans.

The Designing of Mathematics Lesson Plan

Mathematics teachers regard the lesson plan as the resource that point out what and how to teach mathematics effectively in the classroom. A lesson plan is considered to be a teachers' detailed description for teaching a particular lesson. It is the essential tool for effective teaching and important guidance for optimal learning (Ika et al., 2017; Iqbal et al., 2021). It is also "provide teachers with a framework for conceptualizing, designing and delivering instruction" (Ozogul & Sullivan, 2009, p.393). A lesson plan is designed by teachers in daily basis within a class time of one to two period (Ndihokubwayo et al., 2020) before implementing in the class. Through the lesson plan, teachers prepare a systematics learning process (Zainil et al., 2020), thereby, they consider every section of it to have an influence in the teaching and learning.

A well-designed mathematics lesson plan provides evidence that teachers are expertise in incorporating different knowledge components that associated with it. Researchers have identified that teachers' knowledge of content, knowledge of learners and knowledge of curriculum are involved during lesson plan preparation (Sawyer, 2018). Lesson plan designing is also observed to motivate teachers' thinking of learners' specific needs (Diem & Thathong, 2019), so that appropriate learning styles and methods are chosen (Kubilinskiene & Dagiene, 2010). It is also inspired teachers to become innovators and offer new ways of teaching, to use different teaching aids and strategies so that the learners can achieve better result. In this way the lesson plan helps mathematics teachers to improve their confidence in teaching as it assists them to know what they are going to do during the lesson, and hence, get prepared to confront with unexpected issues.

Researchers not only consider the lesson plan to be a guideline for effective performance of the classroom practice but also the recommendation for the development of professional knowledge for teaching (Rusznyak & Elizabeth, 2011). During the teachers' education program, pre-service teachers are introduced to the complexities of teaching in which planning and practicing is amongst. According to Ika et al. (2017) "lesson plan could be a guide to make an optimal learning, with clear steps and part of a unitary form coverage learning material that has been determined for a specific time period" ... Thus, mathematics teachers are also expected to be well equipped and experienced the skill for designing effective lesson plan.

Much has been written about the designing of the effective lesson plan in relation to effective mathematics teaching. This is because better mathematics learning is connected with strong confidence, enjoyment and motivation. Hence, a good lesson plan needs to be effective and excited in order to produce better outcomes (Ndihokubwayo et al., 2020). According to Milkova (2016), a good classroom environment is observed when the lesson plan is well constructed. Thus, the proper designing of mathematics lesson, results to confidently delivery of mathematics lesson.

Relation Between PCK of Mathematic Teachers and Mathematics Teachers' Lesson Plan

Numerous studies revealed that, better awareness of PCK for mathematics teachers is a pre-requisite for better preparation of the lesson, hence results in the effective performance of teaching. Karim and Danaryanti (2020) revealed that planning of the lesson is regarded as among the aspects used to measure PCK ability of mathematics teacher. PCK provides an opportunity for mathematics teachers not only to consider the knowledge of the subject matter to be taught, but also the knowledge that allows the transformation and presentation of mathematical content in a way that learners can easily understand (Danisman & Tanisli, 2017; Karim & Danaryanti, 2020; Shulman, 1987). Therefore, teachers can organize and incorporate different components of PCK required for effective mathematics teaching and

learning during lesson plan designing. Rusznyak and Elizabeth (2011) stated that among the ways of promoting the construction of PCK by students' teachers is to consider the connection of the component parts of PCK during lesson planning.

The impact of better understanding of PCK for mathematics teachers is in the achievement of learning which is measured by students' knowledge, skill, and attitude. According to (Callingham et al., 2016; Cueto et al., 2017; Das, 2019; Taşdan & Çelik, 2016), teachers with high PCK was found to be successful in achieving the goals of teaching mathematics, efficiently in effecting students' attitude towards mathematics and capable in providing a meaningful mathematics learning for students. This imply that teachers are able to choose appropriate competences and realistic goal when planning the lesson. Also, to design the competences which are measurable, observable and reflect the real life, provided those teachers consider knowledge, skills and attitude to be important.

It is of the essence for mathematics teachers to design the lesson plan based on students' demands and talent. Teachers should always think of their students and prepare the lesson by focusing on their needs and their natural ability in learning. Therefore, having a better quality of PCK, enable teachers to deeply consider the learners and their ways of learning. Consequently, the case of consider students' individual difference (Zainil et al., 2020) in terms of the level of understanding, different cognitive levels (Ball, 2000) different learning style, difficulties in learning, pre-conception and misconception of students (Shulman, 1987) are regarded as priority to these teachers during lessons' designing. In this way, teacher with a better quality of this knowledge of PCK would be flexible in using range of teaching strategies and teaching methods, effective in the selection of various appropriate teaching material and resources (Shohani et al., 2015) together with activities that can fit with a topic to be taught and the learners learning style. Thus, teachers will be able to capture an understanding of the lesson to all students in the class and hence provide equal opportunity in learning.

In the designing of the lesson plan, teachers should always consider the assessment for learning. Mathematics teachers used assessment for learning to observe the progress of students throughout the process of teaching so as to improve learning. This process can help the teacher to identify the strengths and weaknesses of students, and provide assistance to them. (Iqbal et al., 2021) Mathematics teachers with improved PCK are supposed to have different kinds of learning activities planned for students so that they are able to assess them during the ongoing process of teaching the lesson. It may happen for the teacher to makes a plan that is difficult for students on that particular lesson. Therefore, the assessment for learning will allow the teacher to be flexible and to change the plan accordingly.

Teachers' understanding of the content is not sufficient to make them competent in designing good lesson plans (Ma'rufi, 2015). This is because content knowledge is only associated with what to teach. There must be other knowledge beyond that of content which associates with how to teach. Teachers' consideration of PCK during the lesson planning is appropriate in helping them to incorporate different elements effectively within mathematics lesson. Thus, the designing of appropriate activities in all sections of the lesson planning is crucial for the leaner to deeply experience learning. These activities include teaching, learning and assessment for learning in all steps starting with introduction, building of new knowledge, reinforcement, reflection, summery and conclusion. In this way PCK competences should be prioritized and must be possessed by the mathematics teachers prior to the lesson plan preparation. And it became effective in determining the uniqueness construction of the lesson for the teachers.

Numerous studies about PCK in mathematics focused more on observing issues related to conceptual understanding (Taşdan & Çelik, 2016), theories in extending the framework of this knowledge (Martinovic & Manizade, 2017) , examining teachers' knowledge (Danisman & Tanisli, 2017) or observing the effectiveness of this knowledge on students' outcomes (Callingham et al., 2016; Cueto et al., 2017). Less attention is given to these mathematics teachers to understand the impact of their PCK in relation to the designing of their lesson plan. Several studies were also conducted more on pre-service teachers (Muhtarom et al., 2019; Yarkwah et al., 2020) despite the fact that the challenges on the participation and performance of mathematics are also observed to the teachers who are already at the work of teaching. Therefore, the need to investigate PCK for these mathematics in-service secondary teachers in their designing of the lesson plan is of great importance. To the best of our knowledge, no any study that was reported to be conducted in Unguja-Island of Zanzibar- Tanzania regarding mathematics teachers' PCK, more on those with associated their competences in the lesson plan designing. Lack of this report on the extent of teachers' knowledge led this study to be essential.

The study investigates on how the competences of mathematics teachers' PCK impact the designing of effective lesson plans. It bridges the gap between teachers' PCK competences and designing of the lesson plan, to ensure effective teaching and learning activities in the classroom practice. The study used mathematics content knowledge (MCK), knowledge of representation and strategies (KRS), and knowledge of learners (KL) as the components of PCK. As such, it aimed at answering the following questions:

- i. How do mathematics teachers' PCK competences impact their lesson plan designing in accordance with the LPFG?
- ii. How do mathematics teachers implement their PCK competences in designing their lesson plan for effective teaching?

Methodology

Research Designing

Mixed-method research design were employed of which both elements of qualitative and quantitative approaches were combined to provide a more comprehensive understanding of the research questions. This method was approved to be effective to make the study more suitable (Creswell, 2014).

Sample and Data Collection

Twelve in-service secondary mathematics teachers were purposively sampled from 12 public secondary schools. These schools were selected randomly from three regions of Unguja-Island of Zanzibar. These teachers were sampled based on the level they teach in their respective schools of which level three was the target in this study. Four of these teachers were females while eight were males. The age range of these teachers were from 27 and 54 with teaching experience from 4 to 35 years. Out of these participants six hold diploma, five hold bachelor degree and one hold master. Despite the fact that these teachers teach mathematics, four of them did not specialize this subject in their teacher's education development program. These teachers teach the subject because of the deficiency of mathematics teachers in the secondary schools of Zanzibar. However, these teachers received training in the special program which assist them in their mathematics content knowledge. All these teachers were involved in this study after accepting and signing the consent form.

The review of the mathematics teachers' lesson plan documents and teachers' interviews were used as the instruments in the data collection. Each of the 12 teachers provided three documents of the lesson plan which were designed to three different period before the lesson, to make the total of 36 lesson plan documents. The lesson plan documents were reviewed in accordance with LPFG and were designed based on the specific topic of relation and function (see appendix D). These two topics are closely related and they are taught during the first twelve weeks of the year, hence it enables the researchers to be consistent in the assessment of the topics.

Corresponding to that, each of the twelve teachers was interviewed in order to explore more information about their consideration in their lesson plan designing. The interview prepared was semi structured and composed of 17 questions (see appendix C). The first five questions related to the first research question, while the rest used to answer the second research question. Each teacher was interviewed once after the collection of their lesson plan documents, and was conducted for approximately 45 minutes. The interviewee was given chance to add more explanation connected to the provided question. The interview questions were reviewed by educational experts before presented to the teachers. Also, researchers insured the credibility and trustworthiness during these teachers' interviews.

Analyzing of Data

The data were analyzed in both quantitative and qualitative mode. In responding to the first research question, descriptive analysis was used where by the consistency of the percentage average of the occurrence of each section between the collected lesson plan and interview were determined. Also, the overall percentage average of the occurrence of all sections in the designed lesson plan in both the reviewed documents and interview were checked. Additionally, the percentage average of the occurrence of the lesson development section based on time, teaching activities, learning activities and assessment activities for both the reviewed documents of the lesson plan and the interview were determined. Based on this analysis, 50% and below were assigned as (poor) percentage average of occurrence of the section, while 90% and above were considered as (excellent). Other categories assigned were 60% (less sufficient), 70% (satisfactory), and 80% (very good). The percentage average was assigned to be above the satisfactory level to indicate the consistency grouping of the sections in the teachers DLP and hence, the sufficient competences of mathematics teachers' PCK to impact their lesson plan designing in accordance with the LPFG. The analysis techniques were adapted from Shimmel and Columba (2016), Zainil et al. (2020).

Furthermore, the criteria related to the implementation of PCK competences in the DLP for effective teaching were analyzed in each of the five aforementioned sections in connection to the indicators for PCK abilities (see appendix B). These criteria were categories as bad (0), limited (1), developed (2), achieved (3) and exemplary (4) depending upon how far the designed section meet the criteria. The percentage number of occurrence of each category in each section was identified. Thus, bad category reflects to the sections where no criteria associated with the indicators of PCK abilities are encountered, limited is assigned to those criteria with limited evidence of implementation, developing regard to those criteria with some evidence of implementation, achieved deal with criteria which is achieved at a satisfactory level and exemplary relate to the criteria with extensive evidence of implementation. If the criteria in the particular section are encountered, it indicated that teachers have PCK ability in the designing of that corresponding lesson plan section, and hence their PCK competences is identified to be in that categorical level. PCK abilities indicators was adapted from (Aminah & Wahyuni, 2019). In addition to the table and graph form of representing the result of these analysis, the detail description was also used.

The authors consider the analysis of the data as reliable given that it was adapted from Shimmel and Columba (2016), Zainil et al. (2020) and Aminah and Wahyuni (2019) and was done based on the LPFG from the Ministry of education. Moreover, the codes were assigned regarding the number of times the lesson plan section occurred in each of the 36 lesson plans collected from 12 teachers together with 12 interview responses. And they were ensured as reliable as they were revised and tested by three different experts. One is a consultant in the education program and educational leadership. The remaining others are professionals in the teacher professional development institution. All three experts arrived to the nearly same results.

Results

i. How do the mathematics teachers' PCK competences impact the lesson plan designing in accordance to the LPFG?

Figure 1 shows the percentage average of occurrence of the lesson plan sections designed by mathematics teachers in their review documents and interviews. The result indicated all twelve teachers were given priority to the sections such as regular information, main topic, subtopic and specific objective. Hence, the percentage average of occurrence of these sections is 100% (excellent). The result also shows other sections have the same but not 100% percentage average of occurrence between the reviewed documents and interviews. This indicated that neither all sections were included in the teachers' designed documents nor mentioned during interview. Example of these sections are reference 75% (satisfactory), building of new knowledge 75% (satisfactory), teacher assessment 83.33% (very good), student work 66.67% (less sufficient) and remarks 83.33% (very good). The differences in the percentage average of occurrence were observed between review documents and the interview in the sections such as the competence, which carried 66.67% (less sufficient) in the lesson plan documents, but only 50% (poor) in the interview and other sections such as teaching and learning materials, introduction, reinforcement, reflection and summary.

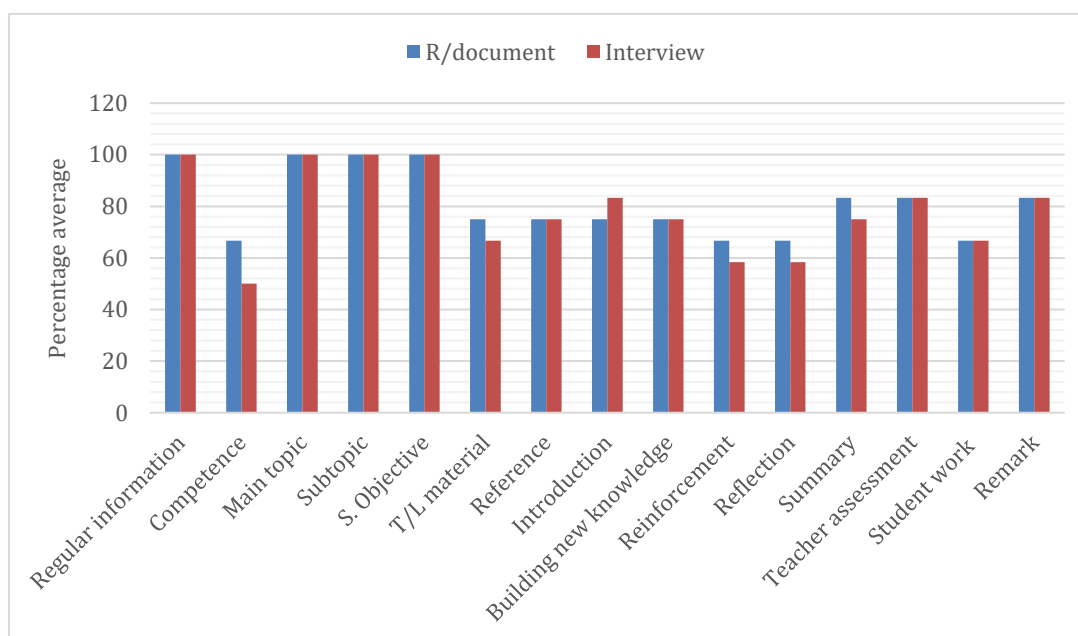


Figure 1. The Percentage Average of the Lesson Plan Indicating Sections Occurred in the Review Documents and Interview

Table 1 presents the overall percentage average of the occurrence of all sections obtained between the review documents and interview. The result indicate that teachers are at the satisfactory level in following the LPFG when designing their lesson plan. However, the overall percentage average of occurrence appeared to be less sufficient regarding the sections which did not attend in all documents of the lesson plan as well as mentioned in interview. These include all sections except that of regular information, topic, subtopics and specific objectives.

Table 1. The Overall Percentage Average of the Sections Occurred between the Review Documents and Interview

	R/documents	Interview	% Average
If all sections are included	81.11%	78.33%	79.72%
If some sections are excluded	72.22%	67.59%	69.91%

Figure 2 demonstrate the percentage average of occurrence of the lesson development steps observed in terms of time, T/L activities and assessment activities. The result revealed the instability in the percentage of occurrence of these steps in their DLP. The focus in many documents were on teaching and learning activities rather than time and assessment activities. Similar results obtained during interview when the same teachers were revealed about their

consideration in completing the lesson development section, they seemed to ignore about the time taken for each activity as well as assessment activities.

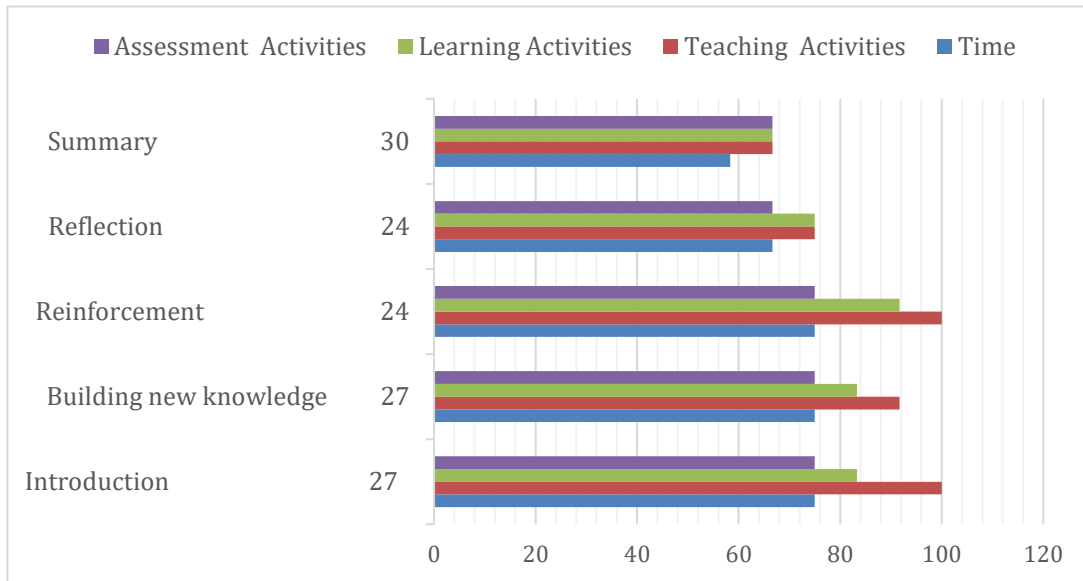


Figure 2. The Percentage Average of the Lesson Development Steps Characterized in Terms of Time, T/L Activities and Assessment Activities

ii. How do mathematics teachers implement their PCK competences in the designing their lesson plan for effective teaching?

Table 2 summarize the results obtained from the reviewing of 36 lesson plan documents and 12 teachers' interviews. Each section of the DLP was found to be in more than one category. The number (percentage) appeared in the categories indicate the number in which the section encountered the criteria of that particular category. However, some other areas show limited evidence, some start to develop and some have achieved the target, though, none of them have reached the exemplary stage. In most cases the result obtained in reviewing of the lesson plan documents agreed with the one obtained in the interviews, hence, they appeared to be in the same category.

Figure 3 summarize the result of the categories which appear the most in each section of the DLP. The sections such as competence, T/L materials, reinforcement reflection and summary were observed to be dominated by the developing stage in both the review documents and interviews, while introduction section was in limited stage. The specific objectives sections found more to be in achieved standard in the review documents while the developing stage during interviews. The reference section was more pronounced in the limited stage during the reviewing of the documents whereas the achieved standard during the interviews. And the step of building of new knowledge was more found in the limited and developing stage during the reviewing of the documents and interviews respectively.

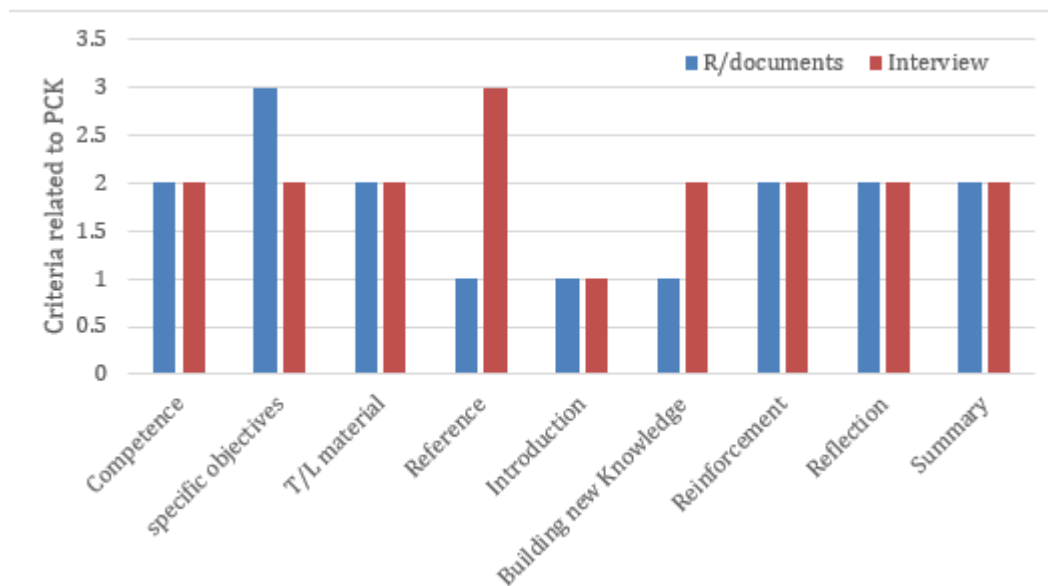


Figure 3. Implementation of PCK Competences Observed in the Lesson Plan Sections in Terms of Pre-Determined Criteria

Table 2. Summary of the Implementation of PCK Competences in the DLP Based on Determined Criteria for Better Plan of the Sections Jointly with PCK Ability

Sections	R/Documents						Interviews					
	N	Bad (0)	Limited (1)	Developing (2)	Achieved (3)	Exemplary (4)	N	Bad (0)	Limited (1)	Developing (2)	Achieved (3)	Exemplary (4)
		N (%)	N (%)	N (%)	N (%)	N (%)		N(%)	N (%)	N (%)	N (%)	N (%)
Competence	36	12(33.33)	6 (16.67)	18 (50)	-	-	12	-	5(41.67)	7(58.33)	-	-
specific objectives	36	-	-	9 (25)	27 (75)	-	12	-	3((25)	9 (75)	-	-
T/L materials	36	9(25)	2(5.55)	25(69.44)	-	-	12	-	2(16.67)	10(83.33)	-	-
Reference	36	9(25)	21(58.33)	6(16.67)	-	-	12	-	1(8.33)	2(16.67)	9(75)	-
Introduction	36	9(25)	24(66.67)	3(8.33)	-	-	12	-	8(66.67)	4(33.33)	-	-
Building new Knowledge	36	9(25)	21(58.33)	6(16.67)	-	-	12	-	3(25)	9(75)	-	-
Reinforcement	36	12(33.33)	3(8.33)	21(58.33)	-	-	12	-	4(33.33)	8(66.67)	-	-
Reflection	36	12(33.33)	3(8.33)	21(58.33)	-	-	12	-	3(25)	9(75)	-	-
Summary	36	6(16.67)	6(16.67)	24(66.67)	-	-	12	-	2(16.67)	10(83.33)	-	-

Further analysis was done in each section of the DLP of which some common challenges related to the criteria in the DLP with the connection to the indicators of PCK abilities were observed to both the reviewed documents and the interview. This result was categorized in two groups:

a) *The results based on particular section:*

Table 3. Observed Challenges in Each Section of the Teachers' DLP

Section	Challenges Observed
Competence	<ul style="list-style-type: none"> The use of general statement which denied the case of being measurable/observable. Example: "The student should be able to use mathematical knowledge, skills and concept to solve real life related problem" (Step function) Did not reflect the real life, and hence seemed to end up in the classroom Example: "at the end of the lesson student should be able to demonstrate the representation of the relation by using arrows related with member of sets" (Relation) Consider only cognitive domain with less intention on the psychomotor and affective domain. Example: "At the end of the topic students should be able to draw graph relation" (Relation)
PCK abilities indicators	Teachers indicate not good enough ability to allow the students opportunity to think critically and become creative.
Specific objectives	<ul style="list-style-type: none"> Confusion between the objective and competences which lead the objective not be specific, measurable and achievable, hence not realistic. Example: "To think critically and logically of mathematically interpreting and solving problem" (Inverse of function) "After classroom pupils should be able to solve problems in daily life" (Relation) Measured student low level of thinking, hence, neglected the students' opportunity to be actively engaged in creativity and critical thinking Example: "At the end of the lesson student should be able to demonstrate the relation pictorially" (Relation)
PCK abilities indicators	Teachers indicate limited ability of focusing to the critical points of the topic and hence they fail to rely on the essential component of the topic. Moreover, they lack student opportunity to think critically.
T/L materials	<ul style="list-style-type: none"> Not good enough extension of the material beyond chalk and board The choice of teaching and learning materials was insufficient to enhance the achievement of all objectives Example: they use of single T/L materials to achieve all specific objective The choice of T/L materials is not realistic in terms of the student demand and content Example: the teachers intended to teach inverse of relation (or inverse of function) but instead, they used the illustration of relation (or function) Confusion between T/L materials and teaching activities Example: library search
PCK abilities indicators	Teachers indicate little ability to link the material with each specific objective and with T/L activities. Also, they indicate little consideration to use the material to capture learner's diversity in terms of the level of understanding. They also indicate not good enough consideration of using the material or challenge learners' cognitively.
Reference	<ul style="list-style-type: none"> Limit the reference to the text book Example: Most of the text book written is or Did not consider other source of information
PCK abilities indicators	Teachers indicate little abilities to select appropriate text book and suitable references for the topics and the learners.
Introduction	<ul style="list-style-type: none"> No activities were observed to capture student prior knowledge, instead teachers jump to the next step of building new knowledge Example: "To ask students to explain the concept of step function" (step function) "To ask student to state all the condition on how to find the domain and range of relation" (relation) "To introduce the subtopic to the student" (inverse of function) Did not intend to familiarize the students with the new topics and also to stimulate students' interest Example "I will guide students to solve problem. (The graph of inequality function) Made revision of previous lesson instead of testing pre-existing knowledge Example "To brainstorm students about the previous lesson" (Relation) "I will make correction of the previous lesson" (Function)
PCK abilities indicators	Teachers indicate little abilities to link previous knowledge with the current one. Also, they indicate little ability to identify students' pre-conception

Table 3. Continued

Building of new knowledge	<ul style="list-style-type: none"> • Incorporate single activity to achieve all specific objectives. • The designed single activity is insufficient to capture students' attention • The designed activities were not clearly observed in some documents (not specified) • Mismatched between the teaching and learning activities. • No multiple representations in building up the concepts • No connection between T/Material with T/L activity
PCK abilities indicators	Teachers indicate little abilities to use various strategies and to facilitate different learning activities to meet the students' difficulties. Also, they indicate little abilities in teaching the concepts using multi-representation to capture learners' differentiation in the level of understanding. Also, little abilities indicated in the use of teaching aids and real-life example to motivate students learning.
Reinforcement	<ul style="list-style-type: none"> • No varieties of activities to captures learners' multiple intelligences • Stuck to a single strategy as the one used in previous steps • Miss the logic of the step in a way that it covered new specific objective which were not mentioned in the step of building of new knowledge. <p>Example: The document observed to <i>covered the concept of properties of step function</i> in building of new knowledge and introduced a concept <i>on how to draw graph of step function</i> (Step function)</p>
PCK abilities indicators	Teachers indicate little ability to facilitate the student -students interaction and allow students experience learning in new ways. They also indicated little ability in organizing the lesson to cover the intended objectives.
Reflection	<ul style="list-style-type: none"> • Confused between this step with the summary step • Some documents skip this step of reflection • Introduce knew specific objective which were not taught in the previous steps <p>Example "I will guide student on how to state domain and range" (the graph of inequality function)</p>
PCK abilities indicators	Teachers indicate little ability to provide examples that overcome students misunderstanding and to respond to the students' error in their learning process. Also, they indicate less ability to reflect to the objectives or competence.
Summary	<ul style="list-style-type: none"> • No activities that incorporate students to summarize the lesson <p>Example: "Summary the lesson ask student to list all important point" (relation)</p> <ul style="list-style-type: none"> • Not activities provided to allow students to ask or seeking clarification of unclear points <p>Example: "To give out main point of the lesson which is very necessary to memorizes" (inverse of function)</p> <p>"I will lead student to summarize the lesson"</p>
PCK abilities indicators	Teachers indicate little abilities to response to the students' errors and to provide opportunities for students to ask

b) General results based on all five steps of the lesson development:

The factors of time were not considered as an important to be included in the lesson development. The result show that there were some documents which did not include time in either step of the lesson development. While others, allocated time in unrealistic form. In this case, teachers were found to allocate more or less time than the required duration in the steps. For instance, *in 80 minutes lesson period, teachers regard the building of new knowledge in 10 minutes, 20minutes for the reinforcement step and 30 minutes for reflection steps. It was also found that out of 80 minutes, teacher allocated time using 70 minutes only.*

Teachers were also reported to have confusion on the actual meaning of assessment activities. Most of the documents were reported to provide question related to the activities of particular step in place of assessment activity. No any activity mentioned by these teachers that assess the understanding and progress of students within the lesson development steps. For instance, the assessment activities in the step of building new knowledge stated "Can the students explain the concept of the inverse of relation?", "Can students listen?", "Are students able to identify and the properties of step function?". Same kind of questions were asked in the assessment activity column of each step in the lesson development and similar were revealed to other teachers' DLP. Generally, teachers were found to copy questions as provided in the syllabus which guide them to find out either the students are able to achieve the steps or not after teaching and learning activities of that particular.

Discussion

Generally, the result of the analysis in the first research question revealed that mathematics teachers are at satisfactory level to include all sections in the DLP. The Ministry of education regards the compliance of the teachers DLP with the LPFG as mandatory. Also, the LPFG provided are well written document to simplify the work for the teachers regardless of the complexity's nature involved in planning. Furthermore, all teachers are qualified at the level of diploma, degree

and master degree, so that they are well trained to design the standard lesson plan. Therefore, it is expected for these teachers to achieve very good or excellence.

When the sections such as regular information, topic, subtopic, specific objective were excluded, the results reduced to less sufficient. This deviation is due to the fact that the excluded sections do not require, creativities nor competences in their designing, and that is why these sections were observed in all documents as well as they were mentioned during the interviews. While major impact of mathematics' teachers PCK competences is expected to be observed within the remaining sections of the lesson plan. The shortage was observed when these sections were inconsistently involved within the lesson plan designing. Also, the instability observed in the designing of the lesson development section regarding its constituent components indicated that mathematics teachers are striving to comply their designed lesson plan in accordance with LPFG.

Some teachers also disclosed during the interviews that they do not regularly write lesson plans in their preparation; mostly they write it when they are inspected by educational administrators. They also declared not to refer to the lesson plan during their classroom teaching. This confirm the work of (Ko, 2012) who asserted that, education is in the period where the formal lesson plan is mostly used to inform administrators of a teacher's curriculum coverage rather than used as a teaching tool during class.

The guideline provided to these mathematics teachers consider every section of it to be important. And for mathematic teachers who reflect their PCK competences during the lesson planning incorporate all these sections effectively to make it comprehensive in the classroom practices. In their result of analyzing the ability of PCK of mathematics teachers based on multiple intelligent, Aminah and Wahyuni (2019) asserted that mathematics teachers who demonstrate high PCK ability are also having high ability in constructing learning device in which the lesson plan is amongst. This is in line with Ika et al. (2017), who revealed that teachers with high quality of PCK are likely to have better skills in preparing and plan of their lesson. In this regard, this study considers mathematics teachers with PCK competences to design the lesson plan by grouping all the sections in accordance with LPFG.

The general results of the second research question show that mathematics teachers PCK competences are in developing stage to design the lesson plan for effective teaching. These results provide impression that teachers have some skill in considering some criteria for better designing of the lesson. However, they mostly designed lesson plan in accustomed way to satisfy the administration.

The designed competences and specific objective observed by these teachers regarded more of content knowledge at the low level of understanding with little implementation of other related PCK knowledge. Also, the confusion in designing these two sections indicated less organization of the lesson plan which may not lead to attain the intended learning outcome. It is evidenced that clear organization of learning objective is the result for the achievement of competences (Zainil et al., 2020).

Little consideration of learner's diversity in terms of the level of understanding are observed to be the result of choosing only one type of T/L materials for all students, limit the text book, provide single activities to serve both objectives as well as uses of single strategies and representation. This indicates insufficient of these mathematics teachers to relate their knowledge of mathematics topics, their knowledge of representation with strategies and knowledge to their learners. Hence mathematics teachers' PCK competences observed not to be in proper level to influenced the quality of the DLP. The result agreed with the idea of Rusznyak and Elizabeth (2011), who revealed that teacher's ability to relate the component of PCK, which are contextual knowledge of learners and content knowledge are the sign of considering learners' diversity.

The lesson development needs to be in organized way. All steps related to this section in its constituent components are supposed to be well presented to see the logical sequency and flow of the lesson. Failure to do so in the result indicate less sufficient understanding of some theory and practices of teaching and learning. It is expected for teachers to provide introduction based on the theory of constructivism, that new knowledge is build based on the prior-knowledge. Also, introductory steps generate motivation and focus the attention of students to actively engaged in the teaching and learning process (Zainil et al., 2020). The case that was also observed by Boikhutso (2010) in his study where by student teachers used to recap to the previous lesson during their introduction of the lesson. In this regards, the teachers' knowledge of learners as components part of PCK in identifying students' pre-conception and misconception is not given priority by these teachers.

All specific objectives are supposed to be covered in the step of building knew knowledge and reinforced using the step of reinforcement using different strategies. Student-student interaction indicate the maturity of teachers in utilizing the competences base curriculum, the case which were not clearly observed in this study. It is also known that, the attention span for students in the lesson takes about 15 minutes, otherwise teachers need to change the task or to break the task into pieces. Because of the single strategies used by teachers in different steps of the lesson development (building of new knowledge and reinforcement), students may fail to concentrate to the entire process of learning.

Assessment for learning is the sign that teachers are checking the progress and understanding of their student within the lesson. However, the use of question appeared in the assessment activity column of this study indicate the

uncertainty of these mathematics teachers in using the component effectively. Also, the result time factor as component in the lesson plan development provides an impression in this study that teachers are not aware that proper management of the lesson is a result of being realistic in time.

Effective teaching is the one in which teachers are very skilled in the content they taught and are capable of explaining the content in a way that students will understand (Ma'rufi, 2015). This can be achieved if these mathematics teachers are able to intertwine different kind of knowledge. The complexity and demanding of lesson planning require the transformation and interpretation of the significant range of knowledge (John, 1993, as cited in Jones & Smith, 1997). Based on the aforementioned observation, this study revealed that most of teachers are struggling to implement the appropriate approach and strategies to achieve specific objectives. Rohaan et al. (2010) revealed that teaching effectively involves teachers understanding of the appropriate approach and strategies based in certain situation. In this view this study has indicate that PCK competences implemented by mathematics teachers to frame the lesson plan for the effective teaching is in developing stage. Teachers need to improve their ability to design deep learning activities to facilitate environment for students to experience learning of mathematics competences. This is supported by the study of (Sieberer-Nagler, 2016) who revealed that teachers are supposed to design active learning environment that offer opportunity for students to practice what they are learning so as to become competence. In this way students will be actively engaged in class and achieve outcome of each step of lesson development. As stated by (Sieberer-Nagler, 2016) this should be observed from the beginning of the lesson and proceed throughout the lesson introduction up to the lesson's closure.

The role of the teachers in the new reviewed curriculum is to facilitate the lesson, but it has been observed that teachers are the one who own the lesson. Lack of student- student interaction in different steps provide impression that teachers still use content base teaching method instead of competences base which is demanded by the curriculum. This is supported by Boikhutso (2010) who asserted that despite the demand of using student-centered approach, most of developing country still use teacher-centered as a standard way of teaching and learning. The result in his study show that most of the activities planned by pre-service are teacher-centered, however others are theoretical learners centered but technically are "learner-directed-teacher-initiated". Hence, teachers fail to provide many interactive activities among students in their lesson plan designing. Consequently, they deny the students opportunity to own their lesson, and hence fail to engage in creativities and critical thinking. Thus, PCK in terms of knowledge representation and strategies together with teachers' knowledge of learners become limited.

Conclusion

The investigations' results of this study lead us to the conclusion that mathematics teachers are not consistent to include some sections in their designing of the lesson plan. This indicate that their PCK competences are not good enough to impact the design of the lesson plan in accordance with the LPFG. Also, the implementation of PCK competences in the designing of the lesson plan for effective mathematics teaching was observed to be in developing stage, as some criteria which related to it were not clearly observed in the designed lesson plan.

Recommendations

The role of the lesson planning in the aspect of education is great. According to Boikhutso (2010), the use of lesson plan is on improving classroom teaching, pedagogical practices, aspects of participation, intervention, achievement and learning outcomes, hence whatever it takes to school improvement should consider the plan and preparation of the lesson. And all of these can be achieved by the teachers with high competences in PCK. Therefore, it is recommended for in-service mathematics teachers to be provided with training on the implementation of teachers' competences particularly PCK in the lesson plan designing for effective classroom practices. More studies are also recommended to explore the challenges that associate with the designing of competence base lesson plan for effective mathematics teaching practice. Additionally, educational institution should put more effort on improving the quality of teacher education programs, based on the world and curriculum changes.

Limitations

This study was conducted to only 12 mathematics teachers who teach at level three secondary school. It was heavily relied on the review documents of the lesson plan and interviews, and did not go further to check for the implementation of the lesson plan in the classroom practice. It was only considered three lesson plan documents and single interview per teacher. The topics of relations and functions were only the area of concerned in this study.

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Authorship Contribution Statement:

Moh'd: Conceptualizing, designing, analysis and interpreting, writing. Uwamahoro: Interpreting, revision of manuscript, supervision, final approval. Orodho: Revision of manuscript, editing, supervision, final approval.

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Appendices

Appendix A

Lesson Plan Framework Guideline (LPFG)

Lesson plan section					
1	Regular information about the date, class, period, duration, and number of students enrolls in the class and number of students present in the class in terms of gender				
2	Competences in terms of knowledge, skill and attitude				
3	Main topic				
4	Subtopic				
5	Specific objectives				
6	Teaching and learning material				
7	Reference				
8	Lesson development				
	Steps	Time	Teaching Activities	Learning Activities	Assessment Activates
	i		Introduction		
	ii		Building new knowledge		
	iii		Reinforcement		
	iv		Reflection		
	v		Summary		
9	Teacher Assessment				
10	Student work				
11	Remark				

The framework guideline is adopted from (MoE, 1995; MoEVT, 2016)

Table 2: Continued

5 Lesson development			
i.	Introduction	<ul style="list-style-type: none"> -Focus on awareness and familiarization of the topic. -Stimulate student interest or encourage thinking. -Provide activity to capture students' prior knowledge -Provide strategies to capture student's pre-conception and/or misconception Realistic time for each activity 	<ul style="list-style-type: none"> - Ability to link the previous knowledge with the current one. -Ability to introduce the topic and objectives clearly -Ability to use various techniques to motivate students' willingness to learn. -Ability to notice pre-conception -Ability to implementing the learning in accordance with time allocation that is planned. -Ability to monitor the learning progress
ii.	Building of new knowledge	<ul style="list-style-type: none"> -Focus on the key concept of the topic to be taught -designed activities based on specific objectives -Involve activities that capture the attention of many students - time realistic for each activity -Identify strategies that check for students' understanding 	<ul style="list-style-type: none"> -Ability to relate different aspect of mathematics concept. -Ability to select effective teaching methods and techniques to guide students thinking and learning in mathematics. -Ability to use various ways and strategies to meet the difficulties of the students during the lesson. -Ability to facilitate different learning activities which aim to help the students' learning process. -Ability to teach the concepts using multi-representation to capture different level of understanding of students. -Ability to use teaching aids to motivate students learning. - Ability to provide examples to relate the lesson with real life. -Ability to response to the students' error as the stages of the learning process. -Ability to provide opportunities for students to ask. -Ability to implementing the learning in accordance with time allocation that is planned. Ability to monitor the learning progress. -Ability to organize and maintain classroom management

Table 2: Continued

iii. Reinforcement	<ul style="list-style-type: none"> -Involve different students' activities -Include different examples -Student – student interaction is more compare to teacher-student interaction - Realistic time for each activity -Identify strategies that check for students' understanding 	<ul style="list-style-type: none"> -Ability to implement the designed learning activities which aim to help the students' learning process -Ability to use varieties of examples for students in the lesson -Ability to provide appropriate examples for the students to learn -Ability to provide examples to relate the lesson with real life. -Ability to facilitate the interaction of student -students -Ability to response to the students' error as the stages of the learning process -Ability to use teaching aids to motivate students -Ability to provide opportunities for students to ask -Ability to implementing the learning in accordance with time allocation that is planned -Ability Monitor the learning progress. -Ability to carry out follow-up and provide reinforcement -Ability to organize and maintain classroom management
iv. Reflection	<ul style="list-style-type: none"> -Reflect on the competences of the lesson. -Link with specific objectives of the lesson. -Consider to correct students' misconception -Provide method of assessing students - Realistic time 	<ul style="list-style-type: none"> -Ability to implement learning activities which aim to help the students' learning process. Ability to provide activities that reflect to the objectives of the topics. Ability to provide activities that reflect to the competences of the study -ability to provide examples that overcome students misunderstanding -Ability to response to the students' error as the stages of the learning process -Ability provide opportunities for students to ask. -Ability to implementing the learning in accordance with time allocation that is planned -Ability to monitor the learning progress involve students to reflect -Ability to organize and maintain classroom management
v. Summary	<ul style="list-style-type: none"> -Emphasize on the main points of the topic taught -Involve clarification of the unclear concept -Provide activities for integrating the central ideas in a meaningfully. -Provide activities for measuring student understanding. - Time realistic 	<ul style="list-style-type: none"> -Ability to implement the learning activities in accordance with the designed task. - Ability to involve students to summarize -Ability response to the students' error as the stages of the learning process -Ability to provide opportunities for students to ask -Ability implementing the learning in accordance with time allocation that is planned -Monitor the learning progress

The criteria for better planning of the section in the designed lesson plan adapted from the literatures PCK ability adapted from (Aminah & Wahyuni, 2019)

Appendix C:*Interview Question*

The interview questions are prepared to gather information about the PCK of mathematics teachers' competences in designing the lesson plan. The interview was divided into two parts. The first part concerned with the mathematics teachers' PCK competences to comply with LPGF in designing their lesson plan, while the second part was about the implementation of PCK competences in the designing of lesson plan for effective mathematics teaching. The information obtained will help to improve the teaching of mathematics.

Part 1

1. Can you mention the sections that you normally include in your lesson plan?
2. Which of these sections require more skills to design?
3. What do you consider to complete the lesson development section?
4. Do you think is essential to refer to the lesson plan during teaching? Why?
5. How often do you use the lesson plan in the classroom?

Part 2

6. What do you think are the criteria for better designing of the competences in your lesson plan?
7. What do you consider to design specific objectives in your lesson?
8. Do you think is necessary to use T/L materials for your topic? How often do you use T/L materials in your lesson?
9. Are the text books sufficient for your lesson? Why?
10. Why do you think introduction is important in lesson development?
11. How do you design the step of building new knew knowledge? Give example of activities you provide in this step
12. Why reinforcement is important and what is different with building of new knowledge
13. How do you lead reflection in your lesson?
14. Why do you think is important to summarize the lesson before it closures?
15. What do you think are the criteria for better T/L activities in each step of the lesson development?
16. How do you understand about the assessment activities? Do you think is important to reflect to it in each step of the lesson development? Why?
17. How do you ensure that your students have understood what you taught? When, in the lesson, do you check for their understanding? Can you list some strategies you use to monitor?

Appendix D*List of Subtopics Appeared in the Topic of Relation and Function*

Relations	Functions
1. Relations <ol style="list-style-type: none"> i. Find relations between two sets ii. Find relations between members in a set iii. Demonstrate relations pictorially 	1. Representation of a Function <ol style="list-style-type: none"> i. Explain the concept of a functions pictorially ii. Identify functions
2. Graph of a Relation <ol style="list-style-type: none"> i. Draw a graph of a relation represented by a linear inequality 	2. Domain and Range of a Function <ol style="list-style-type: none"> i. State the domain of a function ii. State the range of function
3. Domain and Range of a Relation <ol style="list-style-type: none"> i. State the domain of relation ii. State the range of a relation 	3. Graphic Function <ol style="list-style-type: none"> i. Draw graphs of functions
4. Inverse of a Relation <ol style="list-style-type: none"> i. Explain the Inverse of a relation pictorially ii. Find inverse of a relation iii. Draw a graph of the inverse of a relation 	4. Inverse of a Function <ol style="list-style-type: none"> i. Explain the inverse of a function ii. Show the inverse of a function pictorially iii. Find the inverse of a function iv. Draw a graph of the inverse of a function v. State the domain and range of inverse of functions

The topics were obtained from the mathematics syllabus of secondary schools of Tanzania form I-IV (MoEVT, 2010)

Appendix E

Example of the Lesson Plan Documents Written by One of the Twelve Mathematics Teachers

LESSON PLAN

NAME OF TEACHER: MWEMBELADU S. S.

Day and Date	Class	Period	Time	Number of Students					
				Roll			Presents		
				Girls	Boys	Total	Girls	Boys	Total
16/05/2021	Form III	1+2	70mins	28	40	68	20	32	52

COMPETENCE: use mathematical knowledge and skill to
Solve real life problems.

MAIN TOPIC: RELATION

SUB TOPIC: Graph of a relation

SPECIFIC OBJECTIVES:

- The student should be able to draw graph of
- Relation represented by linear inequalities.
-
-
-

TEACHING AND LEARNING MATERIALS: Coloured chalks, manila paper, markers, mathematical instruments and graph papers.

REFERENCES: Secondary basic math book 3 and the internet.

SUBJECT: BASIC MATHEMATICS YEAR: 2019 TERM: FIRST TERM

LESSON DEVELOPMENT:

STEP S	TIM E	TEACHING ACTIVITIES	LEARNING ACTIVITIES	ASSESS MENT
INTR ODU CTI ON		To brainstorm Students about previous lesson	To give example of relation.	Can student give to do the example.
BUIL DIN G NEV KNO WLE DGE		The teacher lead student through questions and answer to plot a graph of a relation in the Coordinate System.	Student in groups to draw a graph of a linear inequalities	Can student able to draw a graph.
REIN FOR CEM ENT		The teacher lead student to draw a graph of relation individually.	Students individually to draw graphs of linear inequalities	Can student able to draw graphs.
REF LEC TIO N		The teacher guide student to do examples.	Students in groups to do example.	
SUM MAR Y		Teacher guide students to complete class work	Students will do class work.	

Teacher's Assessment:

Students' work: One class work and one home work.

Remarks:

Appendix F

Example of the Lesson Plan Designed in the Topic of Relation and Function

Day Date	Class	Period	Time	Number of students					
				Roll			Presents		
				Girls	Boys	Total	Girls	Boys	Total
Monday 21/06/2021	FIIIA	3+4	80min	28	20	48	25	17	42

Competence: Students should have the ability to think critically and logically on the uses of concept of relations in interpreting and solving the real-life related problems

Main topic: Relations

Sub topic: The concept of relations

Specific objectives: At the end of the lesson, students should be able to:

- i. Describe the concept of relations.
- ii. Find relations between two sets.
- iii. Find relations between members in a set.

Teaching and learning materials: Real objects, colored chalk and board, Manilla sheet and marker pen

References: i. Shichzya. F. D (2002). Basic Mathematics for Secondary Schools. Book Three. *Tanzania Institute of Education, Dar es Salaam: Tanzania.*

ii. TIE (2005). Basic secondary mathematics Book 3. *Educational publisher. Dar es salaam.*

- iii. E. I. A (2005) Zanzibar Basic Secondary Mathematics book 3. MacMillan, Aidan
- iv. Online source

Lesson Development:

	Steps	Time	Teaching Activities	Learning Activities	Assessment Activates
i	Introduction	5	Teacher will guide students to provide various relations involved in real life situation based on their experiences and link them with the mathematical concept of relations	Students will provide various relations involved in real life situation and link with the mathematical concept of relation	Brainstorming
ii	Building of new knowledge	30	By using the real objects (e.g students themselves): i. Teacher will guide students to describe the concept of relations mathematically. ii. Teacher will guide students to use various examples and real-life situation, to perform role play in order to find the relations between two sets (two groups) iii. Teacher will lead students to use various examples and real-life situation to find relation between members in two sets (two groups) iv. Teacher will guide students to demonstrate on how to use relation notation to represent relation	Students will apply teach in turn to describe the concept of relations mathematically. Students in group will perform role plays in order to find the relations between two sets (two groups) Students in group will performed role plays in order to find relation between members of two sets Students individually will practice on how to present relation using relations notation	Discussion Discussion Question and answer Discussion Question and answer Observation
iii	Reinforcement	25	Teacher will provide manilla paper and marker pens to each group of students and guide them: i. to designed more examples involved various relations between two sets ii. to designed more examples on various relations between members of two sets iii. to provide demonstration by using relation notation to represent the designed relation	students in group will designed more examples of relations between two sets students in group will designed more example of relations between members of two sets students will represent the designed relation by using relation notation	Observation and group presentation Observation and group presentation Observation and group presentation
iv	Reflection	15	Teacher will guide students to solve problems constructed based on specific objectives	Students individually will solve problems provided by the teacher	Observation and individual selection presentation
v	Summary	5	Teacher will clarify important points and responds from students' questions	Students will ask questions and seek clarifications on the difficult points throughout the lesson	Brainstorming Question and answer

Teacher Assessment: The lesson was well understood, since the students were able to performed all the task provided by the teacher effectively and interactively to achieve the competences and specifics objectives. (This assessment is done after the lesson)

Student work: students will do homework, exercise 1.1A Basic secondary mathematics

Remark: Teacher will proceed with another sub-topic in the next period.